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19/ENG07/006
Petroleum Engineering
ENG 381

The model for the deformation (y) of a structural element is represented by the expression given in equation (1) $x(x-1)y'' + (3x-1)y' + y = 0$

Given that $y(0) = 0.0005$ and $y'(0) = 0.0005$, applying Leibnitz. Maclaurin method

a) Obtain the power series solution of the model up to and including the term in n^7

b) Estimate the appropriate deformation when $x = 5, 8$ and 10

c) With the aid of a mathlab mfile program, plot the response of the structural element for $0 \leq n \leq 10m$.

Solution

Using Leibnitz method

$x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2-x)y'' + (3x-1)y' + y = 0$

$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{2!}2y^n + (3x-1)y^{n+1} + n(3)y^n + y^n = 0$

$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$

$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^n + y^n$

$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$

$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2+2n+1)y^n = 0$

When $x = 0$

$(2n(0) - n + 3(0) - 1)y^{n+1} + (n^2+2n+1)y^n = 0$

$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$

$\times(n+1)y^{n+1} = y(n^2+2n+1)y^n$

$y^{n+1} = \frac{(n^2+2n+1)y^n}{(n+1)}$

$(y^{n+1})_0 = \frac{(n+1)\cancel{(n+1)}(y^n)_0}{\cancel{(n+1)}}$

$(y^{n+1})_0 = (n+1)(y^n)_0$

at $n=0$; $y' = 1y^0$

$n=1$; $y^2 = 2(y')_0$

$n=2;\ y^3 = 3y^2 = 3(2)\,y^1 = 6\,(y^1)_0$

$n=3;\ y^4 = 4y^3 = 4(3)(2)y^1 = 24\,(y^1)$

$n=4;\ y^5 = 5y^4 = 5(4)(3)(2)y^1 = 5!\,(y^1)_0$

$n=5;\ y^6 = 6y^5 = 6(5)(4)(3)(2)y^1 = 6!\,(y^1)_0$

$n=6;\ y^7 = 7y^6 = 7(6)(5)(4)(3)(2)y^1 = 7!\,(y^1)_0$

Maclaurin series

$$y = y_0 + x(y_0') + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$$

$$= y^0 + x(y_0') + \frac{x^2}{2!}(2y_0') + \frac{x^3}{3!}(3!\,y_0') + \frac{x^4}{4!}(4!\,y_0') + \frac{x^5}{5!}(5!\,y_0') + \frac{x^6}{6!}(6!\,y_0') + \frac{x^7}{7!}(7!\,y_0')$$

$$= y^0 + x(y^1)_0 + x^2(y^1)_0 + x^3(y^1)_0 + x^4(y^1)_0 + x^5(y^1)_0 + x^6(y^1)_0 + x^7(y^1)_0$$

$$= y^0 + y^1(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall

$y^1 = y^0$

$y_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y^1 = 0.0005$ and $y^0 = 0.0005$, when $x = 5$

$y_5 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 0.0005(97656)$

$= 48.828$

$\simeq 49$

when $x = 8$, $y_0 = 0.0005$

$y_8 = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$= 0.0005(2396745)$

$= 1198.3725$

$\simeq$

$x = 10$

$y_{10} = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 0.0005(11111111)$

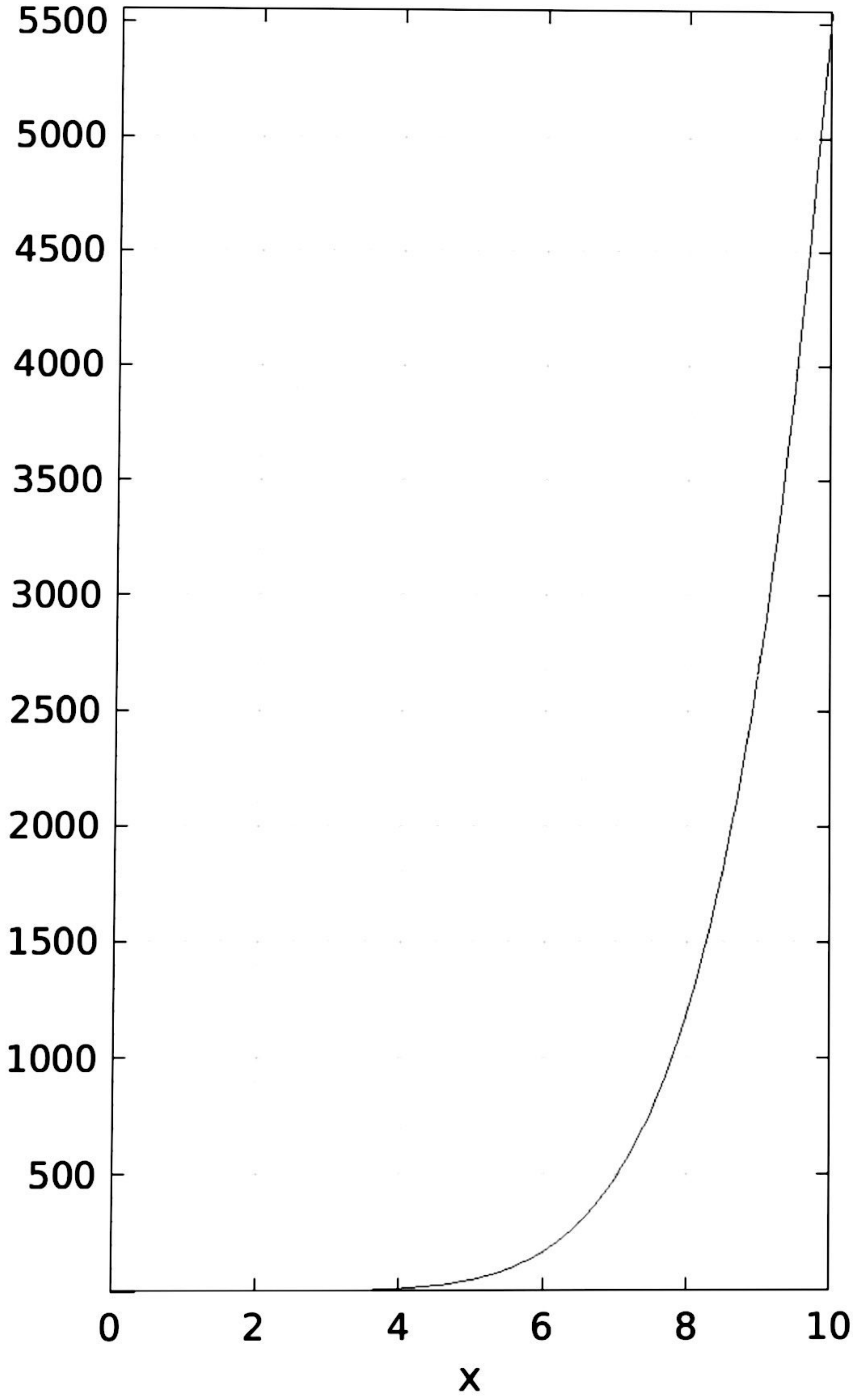
$= 5555.5555$

$\simeq 5555$

Scanned with CamScanner

(c) Command window

```
clc
Clear all
Close all
syms x y
x=0:0.1:10
y=(0.005)*(1+x+x^2+x^3+x^4+x^5+x^6+x^7)
yn = subs(y)
ynn = double(yn)
plot (x,ynn)
x label ('x')
y label ('T')
grid on
grid minor
axis tight.
```


5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
2
4
6
8
10
x