

$$1) \quad x(x-1)y'' + (3x-1)y' + y = 0$$

$$\text{let } u = \frac{1}{x} \Rightarrow y = uy'$$

$$u^{n-1} = y^{n-1}$$

$$u^{n-2} = y^n$$

$$W_2 = (3x-1)y'$$

$$u = y'$$

$$u^n = y^{n-1}$$

$$u^{n-1} = y^n$$

$$W_2 = y$$

$$u = y$$

$$u^n = y^n$$

$$v' = 1$$

$$v' = 0$$

Applying Leibnitz in eq (1) & (2)

$$y^n = y^{n-2} (x^2 - x) + nxy^{n-1} (2x-1) + \frac{n(n-1)}{2} x^2 y^{n-2}$$

$$y^n = x(x-1)y^{n-2} + (2x-1)ny^{n-1} + \frac{n(n-1)}{2}x^2 y^{n-2}$$

$$(1) \quad y^n = y^{n-1}(3x-1) + ny^n(3) + 0$$

$$(2) \quad y^n = y^n \times 1 + 0 = y^n$$

Assuming $x=0$

$$-n(y^{n+1})_0 + n(n-1)y^n + (3n-1)y^{n+1} + 3ny^n + y^n = 0$$

$$x(x-1)y^{n+2} + (2n-1)ny^{n+1} + n(n-1)y^n + (3n-1)y^{n+1} + 3ny^n + y^n = 0$$

Assuming $x=0$

$$-n(y^{n+1})_0 + n(n-1)y^n + (3n-1)y^{n+1} + 3ny^n + y^n = 0$$

$$-(y^{n+1})_0 (n-1) + (y^n)_0 (n(n-1) + 3n + 1) = 0$$

$$-(y^{n+1})_0 (n+1) + (y^n)_0 (n^2 + 2n + 1) = 0$$

$$+(n+1)(y^{n+1})_0 = + (n^2 + 2n + 1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2 + 2n + 1)(y^n)_0$$

when $n=0$

$$(y^0)'_0 = (y^0)_0$$

$$\text{But } (y^1)'_0 = 0.0008 \pi$$

$$(y^1)_0 = 0.0008$$

when $n=1$

$$2(y^2)'_0 = (1+2+1)(y^1)_0$$

$$2(y^2)_0 = 4(y^1)_0$$

$$(y^2)'_0 = 2(y^1)'_0$$

$$= 2 \times 0.0008 \pi = 1 \times 10^{-3}$$

when $n=2$

$$3(y^3)'_0 = (4+4+1)(y^2)'_0$$

$$3(y^3)_0 = 9(y^2)_0$$

$$(y^3)'_0 = 3(y^2)'_0$$

$$(y^3)_0 = 3 \times (1 \times 10^{-3}) = 3 \times 10^{-3}$$

when $n=3$

$$4(y^4)'_0 = (7+6+1)(y^3)'_0$$

$$(y^4)'_0 = 4(y^3)'_0$$

$$= 4 \times (3 \times 10^{-3}) = 0.042 / 12 \times 10^{-3}$$

when $n=4$

$$5(y^5)'_0 = (16+8+1)(y^4)'_0$$

$$(y^5)'_0 = 5(y^4)'_0$$

$$= 5 \times 0.012 = 0.06$$

when $n=5$

$$(y^6)'_0 = 6(y^5)'_0$$

$$= 6 \times 0.06 = 0.36$$

when $n=6$

$$(y^7)'_0 = 7(y^6)'_0$$

$$= 7 \times 0.36 = 2.52$$

Using Leibnitz Ego

$$y^7 = (y^6)'_0 + \pi(y^6)_0 + \frac{\pi^2}{2!}(y^6)''_0 + \frac{\pi^3}{3!}(y^6)'''_0 + \frac{\pi^4}{4!}(y^6)''''_0 + \frac{\pi^5}{5!}(y^6)'''''_0 + \frac{\pi^6}{6!}(y^6)''''''_0 + \frac{\pi^7}{7!}(y^6)'''''''_0$$

$$y^7 = 0.0008 \pi + 0.0008 \pi + \frac{\pi^2}{2!} \times (1 \times 10^{-3}) + \frac{\pi^3}{3!} (3 \times 10^{-3}) + \frac{\pi^4}{4!} (0.002 \times 10^{-3}) + \frac{\pi^5}{5!} (0.008 \pi) + \frac{\pi^6}{6!} (0.036) + \frac{\pi^7}{7!} (2.52)$$

b) When $x=5$

$$y = 5 \times 10^{-4} (1 + 5 + 25 + 125 + 625 + 3125 + 15625 + 78125)$$
$$y = 5 \times 10^{-4} \times 97656$$
$$y = 48.828$$

When $x=8$

$$y = 5 \times 10^{-4} (1 + 8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152)$$
$$y = 1198.3725$$

When $x=10$

$$y = 5 \times 10^{-4} (1 + 10 + 100 + 1000 + 10000 + 100000 + 1000000) = 5555.5555$$

c) ~~for~~

Command Window

Clear

clc

close all

syms x

$$y = (5 * 10^{-4}) * (1 + x + x^2 + x^3 + 2 * x^4 + x_0 * 5 + x_0 * 16 + x_0 * 19)$$

$$x = (0:10)$$

Yn = Subs(y,x)

Plot(xn, Yn)

x Label ('Structural Element (m)')

y Label ('Deformation')

grid on

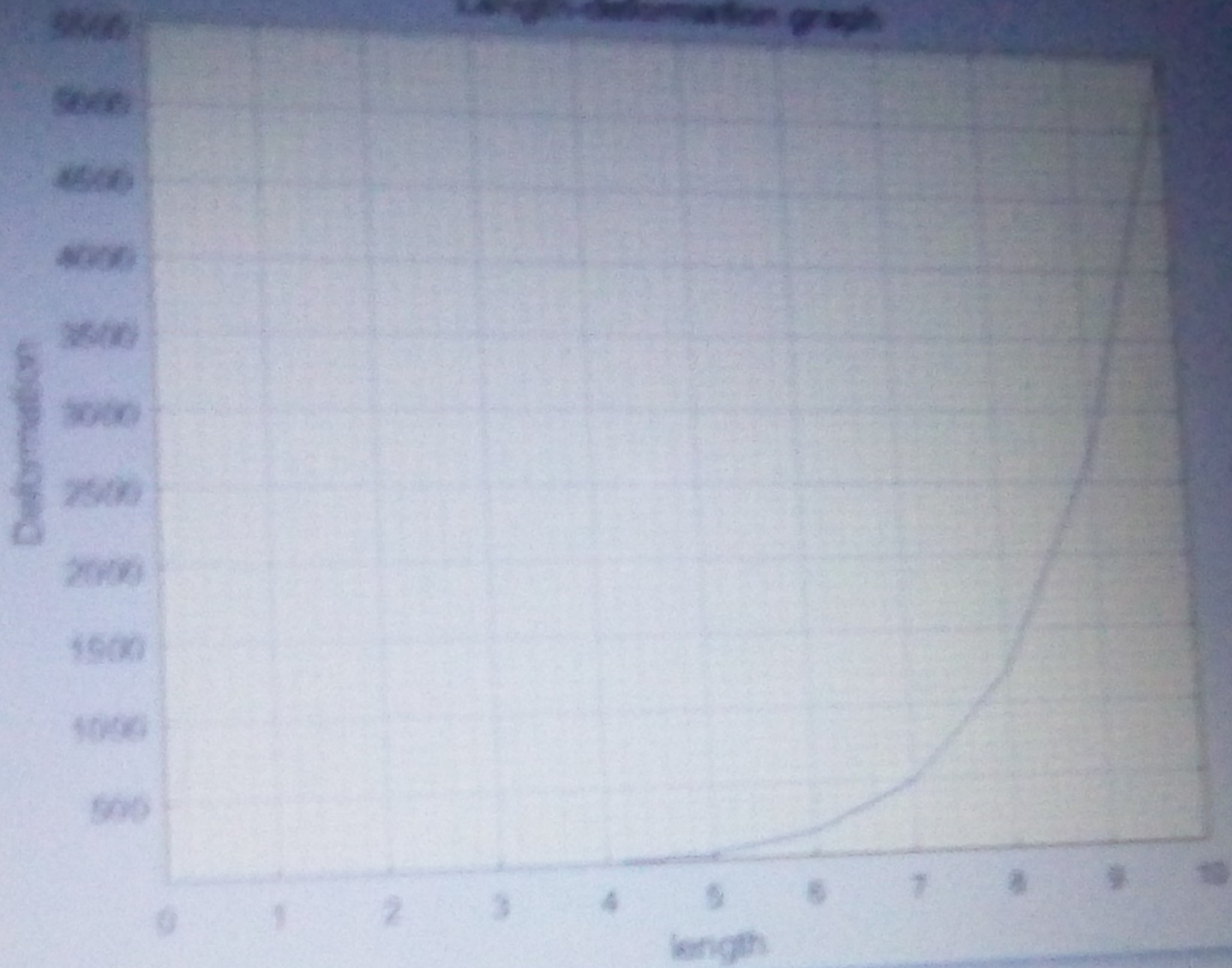
grid minor

axis tight

$$x + \frac{x^6}{6} (y^6)^4$$

$$\frac{x^5}{120} 0.06 +$$

Length-deformation graph



0.0000 0.0001 0.0014 0.0109 0.0482 0.1