

1. Solution

$$(a) \quad x(x-1)y'' + (3x-1)y' + y = 0$$

for the n th derivative,

$$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n$$

$$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$$

at $x = 0$,

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 + 2n + 1) = 0$$

$$-(n+1)(y^{n+1})_0 = -(n^2 + 2n + 1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2 + 2n + 1)(y^n)_0$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

at $n = 0$,

$$(y^{0+1})_0 = (0+1)(y^0)_0$$

$$(y^1)_0 = (y)_0$$

at $n = 1$,

$$(y^{1+1})_0 = (1+1)(y^1)_0$$

$$(y^2)_0 = 2(y^1)_0$$

at $n = 2$,

$$(y^{2+1})_0 = (2+1)(y^2)_0$$

$$(y^3)_0 = 3(y^2)_0$$

$$(y^3)_0 = 6(y^1)_0$$

$$\text{at } n=3, \\ (y^3)'_0 = (3+1)(y^2)'_0$$

$$(y^3)'_0 = 4(y^2)'_0 = 4 \times 6(y^1)'_0 = 24(y^1)'_0$$

$$\text{at } n=4, \\ (y^4)'_0 = (4+1)(y^3)'_0$$

$$= 5(y^3)'_0$$

$$= 5 \times 24(y^1)'_0$$

$$(y^4)'_0 = 120(y^1)'_0$$

$$\text{at } n=5, \\ (y^5)'_0 = (5+1)(y^4)'_0$$

$$= 6(y^4)'_0$$

$$= 6 \times 120(y^1)'_0$$

$$(y^5)'_0 = 720(y^1)'_0$$

$$\text{at } n=6, \\ (y^6)'_0 = (6+1)(y^5)'_0$$

$$= 7(y^5)'_0$$

$$= 7 \times 720(y^1)'_0$$

$$(y^6)'_0 = 5040(y^1)'_0$$

from maclaurin series,

$$y = (y)_0 + x(y^1)'_0 + \frac{x^2}{2!}(y^2)'_0 + \frac{x^3}{3!}(y^3)'_0 + \frac{x^4}{4!}(y^4)'_0 + \dots$$

$$y = (y)_0 + x(y^1)'_0 + \frac{x^2}{2!}(2(y^1)'_0) + \frac{x^3}{3!}(6(y^1)'_0) + \frac{x^4}{4!}(24(y^1)'_0)$$

$$+ \frac{x^5}{5!}(120(y^1)'_0) + \frac{x^6}{6!}(720(y^1)'_0) + \frac{x^7}{7!}(5040(y^1)'_0) + \dots$$

$$y = (y)_0 + x(y')_0 + x^2(y'')_0 + x^3(y''')_0 + x^4(y^{(4)})_0 + x^5(y^{(5)})_0 + x^6(y^{(6)})_0 + x^7(y^{(7)})_0 + \dots$$

$$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots]$$

but $(y)_0 = 0.0005 \text{ m}$ and $(y')_0 = 0.0005$,

$$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b) When $x = 5 \text{ m}$,

$$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 + 0.0005 [97655]$$

$$\therefore y = 48.828 \text{ m}$$

When $x = 8 \text{ m}$,

$$y = 0.0005 + 0.0005 [8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152]$$

$$y = 0.0005 + 0.0005 [2396744]$$

$$\therefore y = 1198.3725 \text{ m}$$

When $x = 10 \text{ m}$,

$$y = 0.0005 + 0.0005 [10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000]$$

$$\therefore y = 0.0005 + 0.0005 [11111110]$$

$$y = 5555.5555 \text{ m}$$

(c) commandwindow

```
clear
```

```
clc
```

```
syms x
```

```
syms y
```

```
x = (0:10);
```

```
y = 0.0005 + 0.0005 * (x + (x.^2) + (x.^3) + (x.^4) + (x.^5) +  
(x.^6) + (x.^7));
```

```
plot(x, y)
```

```
grid on
```

```
grid minor
```

```
xlabel('x')
```

```
ylabel('Structural Deformation')
```