

- Using Leibnitz method,

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{2!}2y^n + (3x-1)y^{n+1} + n(3)y^{n+1} + y^n = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^{n+1} + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^{n+1} + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

when  $x=0$

$$(2n(0)-n+3(0)-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$+(n+1)y^{n+1} = -1(n^2+2n+1)y^n$$

$$y^{n+1} = \frac{(n^2+2n+1)y^n}{(n+1)}$$

$$(y^{n+1})_0 = \frac{(n+1)(n+1)(y^n)_0}{(n+1)}$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

$$\text{at } n=0; y^1 = 1y^0$$

$$n=1; y^2 = 2(y^1)_0$$

$$n=2; y^3 = 3y^2 = 3(2)y^1 = 6(y^1)_0$$

$$n=3; y^4 = 4y^3 = 4(3)(2)y^1 = 24(y^1)_0$$

$$n=4; y^5 = 5y^4 = 5(4)(3)(2)y^1 = 5!(y^1)_0$$

$$n=5; y^6 = 6y^5 = 6(5)(4)(3)(2)y^1 = 6!(y^1)_0$$

$$n=6; y^7 = 7y^6 = 7(6)(5)(4)(3)(2)y^1 = 7!(y^1)_0$$

Maclaurin Series

$$y = y_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$$

$$= y' + x(y_0)' + \frac{x^2}{2!} (2y_0)'' + \frac{x^3}{3!} (3!y_0)''' + \dots + \frac{x^7}{7!} (7!y_0)'''''''$$

$$= y' + x(y_0)' + \frac{x^2}{2!} (2y_0)'' + \frac{x^3}{3!} (3!y_0)''' + \frac{x^4}{4!} (4!y_0)'''' + \frac{x^5}{5!} (5!y_0)''''' + \frac{x^6}{6!} (6!y_0)'''''' + \frac{x^7}{7!} (7!y_0)'''''''$$

$$= y' + y'(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall

$$y' = y''$$

$$y_0 = (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

$$y' = 0.0005 \text{ and } y'' = 0.0005, \text{ when } x = 5$$

$$y^5 = 0.0005 (1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$

$$= 0.0005 (9 + 656)$$

$$= 48.828$$

$$\approx 49$$

$$\text{When } x = 8, y_0 = 0.0005$$

$$y_8 = 0.0005 (1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$$

$$= 0.0005 (2396745)$$

$$= 1198.3725$$

$$\approx 1198$$

$$x = 10$$

$$y_{10} = 0.0005 (1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$$

$$= 0.0005 (1111111)$$

$$= 5555.5555$$

$$\approx 5556$$

C. - Command window

- CLC

- (clear all)

- Close all

- Syms x, y

- x = 0:0.1:10

- y = (0.0005) \* (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)

- Z\_n = Subs(y)

- Z\_n = double(Z\_n)

- Plot(x, Z\_n)

Fig 1.0

- x label ('x')
- y label ('y')
- Grid On
- Grid minor
- axis right

