# ENGINEERING MATHS

1) $x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2-x)y'' + (3x-1)y' + y = 0$ Expanding bracket

We can then assume

$(x^2-x)y'' = w_1$ , $(3x-1)y' = w_2$ , $y = w_3$

$\therefore$ Using Leibnitz theorem $= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} + \cdots$

| $w_1$ | $w_2$ | $w_3$ |
|---|---|---|
| let $u = y^n$ $V = x^2 - x$ | let $u = y'$ $V = 3x-1$ | let $u = y$ $V = 1$ |
| $u^n = y^{n+2}$ $V' = 2x-1$ | $u^n = y^{n+1}$ $V' = 3$ | $u^n = y^n$ $V' = 0$ |
| $u^{n-1} = y^{n+1}$ $V'' = 2$ | $u^{n-1} = y^n$ $V'' = 0$ | |
| $u^{n-2} = y^n$ $V''' = 0$ | | |

$w_1 = (y^{n+2} \cdot (x^2-x)) + n(y^{n+1})(2x-1) + \frac{n(n-1)y^{n,2}}{2!}$

$w_2 = (y^{n+1} \cdot (3x-1)) + n y^n \cdot 3 + 0$

$w_3 = y^n$

$\therefore w_1 + w_2 + w_3 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + n(n-1)y^n + y^{n+}(3x-1)$
$+ n y^n \cdot 3 + y^n = 0$

$= y^{n+2}(x^2-x) + y^{n+1}(2x-n) + y^{n+1}(3x-1) + n(n-1)y^n + ny^n \cdot 3 + y^n = 0$

assuming $x = 0$

$= y^{n+2}(0^2-0) + y^{n+1}(2(0)-n) + y^{n+1}(3(0)-1) + n(n-1)y^n + ny^n \cdot 3 + y^n = 0$

$= -n y^{n+1} - y^{n+1} + y^n(n^2-n) + 3ny^n + y^n = 0$

$\cancel{= y^{n+1}(-n-1) + y^n(n^2 - n + 3n \; y^n + y^n = 0}$

$\cancel{= -y}$

$= y^{n+1}(-n-1) + y^n(n^2 - n + 3n - 1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n-1) = 0$

$y^{(n+1)}(n+1) = y^n(n^2+2n+1)$

$y^{n+1} = y^n(n+1)$ re-occurence relation

$(y^{n+1})_0 = y^n(n+1)_0$

$(y'')_0 = 0.0005$

$(y')_0 = 0.0005$

when $n = 0$ ! $(y^{(0+1)})_0 = (y^{(1)})_0 (0+1)$

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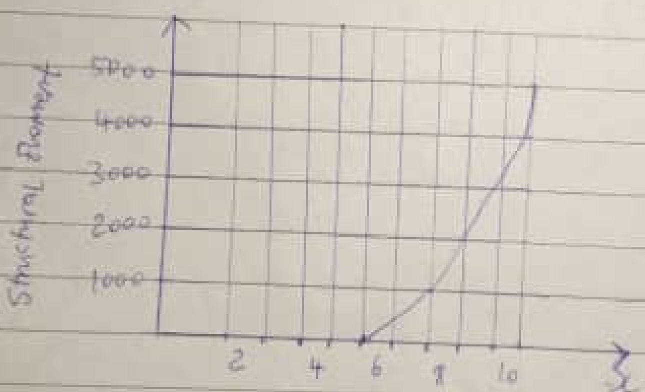
Close all
x = 0.001:10
y = (0.0005 * (x+1)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7)
    * (0.0005)
yn = Subs (Y)
    plot (x, yn)
    axis tight
    grid on
    grid minor
    x label ('M')
    Y label ('Deflection')
```

Sketch

Graph of Structural Element Against x



$(y')_0 = (y^0)_0 = 1$

when $n=1$: $(y^{(1+1)})_0 = (y')_0\,(1+1)$

$(y^2)_0 = (y')_0\, 2 = 2(y^0)_0$

when $n=2$: $(y^{(2+1)})_0 = (y^2)_0\,(2+1)$

$(y^3)_0 = 3(y^2)_0 = 6(y')_0$

when $n=3$: $(y^{3+1})_0 = (y^3)_0\,(3+1)$

$(y^4)_0 = (y^3)_0\, 4 = 24\,(y')_0$

when $n=4$: $(y^5)_0 = (y^4)_0\, 5 = 120\,(y')_0$

when $n=5$ $(y^6)_0 = (y^5)_0\, 6 = 720\,(y')_0$

when $n=6$ $(y^7)_0 = (y^6)_0\, 7 = 5040\,(y')_0$

using Macleorin Series

$y = (y^0)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \cdots$

$y = (y^0)_0 + x(y^0)_0 + \frac{x^2}{2!}\cdot 2(y')_0 + \frac{x^3}{3!}\, 6\,(y')_0 + \frac{x^4}{4!}\, 24\,(y')_0 + \frac{x^5}{5!}\, 120\,(y')_0$
$+ \frac{x^6}{6!}\, 720\,(y')_0 + \frac{x^7}{7!}\, 5040\,(y')_0$

$y = (y^0)_0 + (x+1) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y')_0$

$\therefore y = 0.0005\,(x+1) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)\, 0.0005$

ii) When $x = 5m$

$y = 0.0005\,(5+1) + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)\, 0.0005$

$\therefore y = 48.828m$

When $x = 8m$

$y = 0.0005\,(8+1) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)\, 0.0005$

$\therefore y = 1198.373m$

when $x = 10m$

$y = 0.0005\,(10+1) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)\, 0.0005$

$\therefore y = 5555.56m$

iii) MATLAB m file

Command window

Clear

clc