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17/ENG02/1048

Assignment Solution

Using leibnitz method

$x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2 - x)y'' + (3x-1)y' - y = 0$

$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{2} 2y^n + (3x-1)y^{n+1} + n(3)y^n + y^n = 0$

$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$

$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^n + y = 0$

$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$

$(x^2-x)y^{n+2} + (2nx-n+3n-1)y^{n+1} + (n^2+2n+1)y^n = 0$

when $x = 0$

$(2n(0)-n+3(0)-1)y^{n+1} + (n^2+2n+1)y^n = 0$

$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$

$-(n+1)y^{n+1} = -1(n^2+2n+1)y^n$

$y^{n+1} = \frac{(n^2+2n+1)}{(n+1)} y^n$

$(y^{n+1})_0 = \frac{(n^2+2n+1)}{(n+1)} y^n$

$(y^{n+1})_0 = \frac{(n+1)(n+1)}{(n+1)} (y^n)_0$

$(y^{n+1})_0 = (n+1)(y^n)_0$

at $n=0$; $y^1 = 1y^0$

$n=1$; $y^2 = 2(y)_0$

$n=2$; $y^3 = 3y^2 = 3(2)y' = 6(y')_0$

$n=3$; $y^4 = 4y^3 = 4(3)(2)y' = 24(y')$

$n=4$ $y^5 = 5y^4 = 5(4)(3)(2)y' = 5!(y')_0$

$n=5$ $y^6 = 6y^5 = 6(5)(4)(3)(2)y' = 6!(y')0$

$n=6$ $y^7 = 7y^6 = 7(6)(5)(4)(3)(2)y' = 7!(y')0$

Maclaurin Series

$$y = y_0 + x(y'0) + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)$$

$$= y^0 + x(y'0) + \frac{x^2}{2!}(2y'0) + \frac{x^3}{3!}(3!y'0) + \frac{x^4}{4!}(4!y'0) + \frac{x^5}{5!}(5!y'0) + \frac{x^6}{6!}(6!y'_0) + \frac{x^7}{7!}(7!y'_0)$$

$$= y^0 + x(y')_0 + x^2(y)_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 + x^6(y')_0 + x^7(y')_0$$

$$= y^0 + y'_0(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall

$y' = y^0$

$y_0 = (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y' = 0.0005$

$y^0 = 0.0005$, when $x = 5$

$$y^5 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$
$$= 0.0005(97656)$$
$$= 48.828 \simeq 49$$

When $x = 8$, $y_0 = 0.0005$

$$y_8 = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$$
$$= 0.0005(2396745)$$
$$= 1198.3725$$
$$\simeq 1198$$

$x = 10$

$$y_0 = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$$
$$= 0.0005(11111111)$$
$$= 5555.5555$$
$$\simeq 5556$$

c Command Window

```
clc
Clear all
Close all
Syms x y
x = 0:0:1:10
y = (0.0005) * (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)
yn = Subs(y)
Yan = double(yn)
  Plot (x, Yan)
  x label ('x')
  y label ('T')
    grid on
    grid minor
    axis tight
```