D) The Solution

7) Obtain Power Series

$x(x-1)y'' + (3x-1)y' + y = 0$

The Equation is rewritten as

$(x-y)y'' + (3x-y)y' + y = 0$

$W_1 = (x-y)y''$

$u = y^2$ $\quad Y = (x-1)x = x^2 - x$

$u^n = y^{n+2}$ $\quad V = 2x-1 \quad , V^3 = 0$

$u^{n-1} = y^{n+1}$ $\quad V^2 = 2$

$u^{n-2} = y^n$

Applying lebnitz Equation

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)}}{2!} v^2 + \frac{n(n-1)(n-2)u^{(n-3)}}{3!} v^3 + \frac{n(n-1)(n-2)(n-3)y^{(n-4)}}{4!} v^4 + \dots$$

$$y^n = y^{(n+2)} \times x(x-1) + n \times y^{(n+1)} \times (2x-1) + \frac{n(n-1)}{2} \times y^{(n)} \times 2 \times 0$$

$$y^{(n)} = x(x-1)y^{n+2} + (2x-1)n y^{(n+1)} + n(n-1)y^{(n)}$$

for $W_2 = (3x-1)y^{(1)}$

$u = y^{(1)}$, $u^n = y^{(n+1)}$, $u^{n-1} = y^n$

$V = 3x-1$ $\quad v' = 3$ $\quad v^2 = 0$

Applying lebnitz theorem

$$y^n = y^{(n+1)} \times (3x-1) + n \times y^{(n)} \times 3 + 0$$

$$y^n = (3x-1)y^{(n+1)} + 3ny^n$$

for $W_3 = y$

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

Applying leibnitz theorem

$y^n = y^n \times 1 + 0 = y^n$

Summing all equations together

$x(x-1)y^{n+2} + (2x-1)n\,y^{(n+1)} + n(n-1)y^n + (3x-1)y^{(n+1)} + 3n\,y^n + y^n = 0$

Assuming $x = 0$

$-n(y^{(n+1)})_0 + n(n-1)(y^{(n)})_0 - (y^{(n+1)})_0 + 3n(y^{(n)})_0 + (y^{(n)})_0 = 0$

$-(y^{(n+1)})_0(n+1) + (y^{n})_0[n(n-1) + 3n + 1] = 0$

$-(y^{(n+1)})_0(n+1) + (y^{(n)})_0(n^2 - n + 3n + 1) = 0$

$-(y^{(n+1)})(n+1) + (y^{(n)})_0(n^2 + 2n + 1) = 0$

$\cancel{\Rightarrow}\ -(n+1)(y^{(n+1)})_0 = -(n^2 + 2n + 1)(y^{(n)})_0$

$(n+1)(y^{(n+1)})_0 = (n^2 + 2n + 1)(y^{n})_0$

When $n = 0$

$(y^n)_0 = (y^{(0)})_0$

but $(y^{(0)}) = 0.0005m$

$y^{(1)} = 0.0005$

When $n = 1$

$2(y^{(2)})_0 = (1+2+1)(y^{(1)})_0$

$2(y^{(2)})_0 = 4(y^{(1)})_0$; $(y^{(2)})_0 = 2(y^{(1)})_0 = 2 \times 0.0005$

$= 1 \times 10^{-3}$

When $n = 2$

$3(y^{(3)})_0 = (4+4+1)(y^{(2)})_0 = 9(y^{(2)})_0$

$(y^{(3)})_0 = 3(y^{(2)})_0$

but $(y^{(2)})_0 = 1 \times 10^{-3}$; $(y^{(3)})_0 = 3 \times 1 \times 10^{-3} = 3 \times 10^{-3}$

When $n = 3$, $4(y^{(4)})_0 = (9+6+1)(y^{(3)})_0 = 16(y^{(3)})_0$

$(y^{(4)})_0 = 4(y^{(3)})_0 = 4 \times 3 \times 10^{-3} = 0.012$

When $n = 4$, $5(y^{(5)})_0 = (16+8+1)(y^{(4)})_0 = 25(y^{(4)})$

$y^{(5)} = 0.06$ $\quad = \underset{6}{25} \times 0.012 = 0.3$

When $n = 5$

$6(y^{(6)})_0 = (25+10+1)(y^{(5)}) = 36(y^{(5)})_0 \Rightarrow 6(y^{(5)})$

~~When~~ $n = 5$

$(y^{(6)}) = 6(y^{(5)})_0 = 6 \times 0.06 = 0.36$

$n = 6$

$n=6$

$7(y^{(7)}) - (36+12+1)(y^{(6)}) = 49(y^{(6)})_0$

$(y^{(7)})_0 = 7(y^{(6)})_0 = 7\times 0.36 = 2.52$

Leibnitz Maclaurins method Equation

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^4)_0$$

$$+ \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^7)_0$$

$$y = 0.0005 + 0.0005x + \frac{x^2}{2}\times 1\times 10^{-3} + \frac{x^3}{6}\times 3\times 10^{-3} +$$

$$+ \frac{x^4}{24}\times 0.012 + \frac{x^5}{120}\times 0.06 + \frac{x^6}{720}\times 0.36 + \frac{x^7}{5040}\times 2.52$$

$$y = 0.0005(1+x) + 5\times 10^{-4}(x^2) + (5\times 10^{-4}x^3) + (5\times 10^{-4}x^4)$$

$$+ 5\times 10^{-4}x^5 + (5\times 10^{-4}x^6) + (5\times 10^{-4}x^7)$$

$$y = 0.0005(1+x) + 5\times 10^{-4}(x^2+x^3+x^4+x^5+x^6+x^7)$$

$$y = 5\times 10^{-4}(1+x) + (5\times 10^{-4}(x^2+x^3+x^4+x^5+x^6+x^7)$$

$$y = 5\times 10^{-4}(1+x+x^2+x^3+x^4+x^5+x^6+x^7)$$

When $x = 5$

$$y = 5\times 10^{-4}(1+5+25+125+625+3125+15625+78125)$$

$$y = 5\times 10^{-4}\times 97656$$

$$y = 48.828$$

When $x = 8$

$$y = 5\times 10^{-4}(1+8+64+512+4096+32768+262144 + 2097152)$$

$$y = 1198.3725$$

When $x = 10$

$$y = 5\times 10^{-4}(1+10+100+1000+100000+10^5+10^6+10^7)$$

$$= 5555\ 555$$

Codes

```
Commandwindow
clear
clc
close all
syms x
Y = (5*10^-4)*(1+x+x.^2+x.^3+x.^4+x.^5+x.^6
    + x.^7)
x = (0:10)
Yn = subs(Y,x)
plot (x,yn)
xlabel ('STRUCTURAL ELEMENT (M)')
Ylabel ('DEFORMATION')
grid on
grid minor
axis tight
```