

D) The Solution

Obtain power series

$$x(x-1)y'' + (3x-1)y' + y = 0$$

The equation is rewritten as

$$(x-y)y'' + (3x-y)y' + y = 0$$

$$W_1 = (x-y)y''$$

$$U = y^2$$

$$U^n = y^{n+2}$$

$$U^{n-1} = y^{n+1}$$

$$U^{n-2} = y^n$$

$$V = (x-1)x = x^2 - x$$

$$V' = 2x-1, \quad V'' = 0$$

$$V'' = 2$$

Applying Leibnitz Equation

$$Y^n = U^n V + n U^{n-1} V' + \frac{n(n-1)U^{n-2} V''}{2!} + \frac{n(n-1)(n-2)U^{n-3} V'''}{3!} + \dots$$

$$Y^n = Y^{(n+2)} \times x(x-1) + n Y^{(n+1)} \times (2x-1) + \frac{n(n-1)}{2} Y^{(n)}$$

$$Y^{(n)} = x(x-1)Y^{n+2} + (2x-1)nY^{(n+1)} + \frac{2}{n(n-1)}Y^{(n)}$$

for $W_2 = (3x-1)y'$

$$U = y, \quad U^n = y^{(n+1)}$$

$$V = 3x-1, \quad V' = 3$$

$$U^{n-1} = y^n$$

$$V'' = 0$$

Applying Leibnitz theorem

$$Y^n = y^{(n+1)} \times (3x-1) + n y^{(n)} \times 3 = 0$$

for $W_3 = y$

$$U = y$$

$$V = 1$$

$$U^n = y^n$$

$$V' = 0$$

Applying Leibnitz theorem

Summing all equations together

$$2(x-1)y^{(n+2)} + (2x-1)n y^{(n+1)} + n(n-1)y^{(n)} + (3x-1)y^{(n+1)} + 3n y^{(n)} + y^{(n)} = 0$$

$$-n(y^{(n+2)})_0 + n(n-1)(y^{(n+1)})_0 - (y^{(n+1)})_0 + 3n(y^{(n+1)})_0 + (y^{(n+1)})_0 = 0$$

$$-n(n-1)(y^{(n+1)})_0 + (y^{(n+1)})_0 (n(n-1) + 3n + 1) = 0$$

$$-(y^{(n+1)})_0 (n-1) + (y^{(n+1)})_0 (n^2 - n + 3n + 1) = 0$$

$$-(y^{(n+1)})_0 (n-1) + (y^{(n+1)})_0 (n^2 + 2n + 1) = 0$$

$$\text{When } n=0$$

$$(y^{(0)})_0 = 0.0005 \text{ m}$$

$$\text{When } n=1$$

$$2(y^{(2)})_0 = 4(y^{(1)})_0; (y^{(2)})_0 = 2(y^{(1)})_0 = 2 \times 0.0005 = 1 \times 10^{-3}$$

$$\text{When } n=2$$

$$3(y^{(3)})_0 = 9(y^{(2)})_0; (y^{(3)})_0 = 3 \times 1 \times 10^{-3} = 3 \times 10^{-3}$$

$$\text{When } n=3$$

$$4(y^{(4)})_0 = 16(y^{(3)})_0 = 4 \times 3 \times 10^{-3} = 0.012$$

$$\text{When } n=4$$

$$5(y^{(5)})_0 = 25(y^{(4)})_0 = 25 \times 0.012 = 0.3$$

$$\text{When } n=5$$

$$6(y^{(6)})_0 = 36(y^{(5)})_0 = 36 \times 0.06 = 0.36$$

$$\text{When } n=6$$

$$7(y^{(7)}) - (36 + 12 + 11)(y^{(6)}) = 2 \times 9(y^{(6)})$$

$$(y^{(7)})_0 = 7(y^{(6)})_0 = 7 \times 0.36 = 2.52$$

Liebnitz MacLaurin's method Equation

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0$$

$$+ \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0$$

$$y = 0.0005 + 0.0005x + \frac{x^2}{2} \times 1 \times 10^{-3} + \frac{x^3}{6} \times 3 \times 10^{-3}$$

$$+ \frac{x^4}{24} \times 6.012 + \frac{x^5}{120} \times 0.06 + \frac{x^6}{720} \times 0.16 + \frac{x^7}{5040} \times 2.52$$

$$y = 0.0005(1+x) + 5 \times 10^{-4}(x^2) - (5 \times 10^{-4} x^3) + (5 \times 10^{-4} x^4)$$

$$+ 5 \times 10^{-4} x^5 + (5 \times 10^{-4} x^6) + (5 \times 10^{-4} x^7)$$

$$y = 0.0005(1+x) + 5 \times 10^{-4}(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

$$y = 5 \times 10^{-4}(1+x) + (5 \times 10^{-4}(x^2 + x^3 + x^4 + x^5 + x^6 + x^7))$$

$$y = 5 \times 10^{-4}(1+x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

When $x = 5$

$$y = 5 \times 10^{-4}(1 + 5 + 25 + 125 + 625 + 3125 + 15625 + 78125)$$

$$y = 5 \times 10^{-4} \times 97656$$

$$y = 48.828$$

When $x = 8$

$$y = 5 \times 10^{-4}(1 + 8 + 64 + 512 + 4096 + 32768 + 262144$$

$$+ 2097152)$$

$$y = 1198.3725$$

When $x = 10$

$$y = 5 \times 10^{-4}(1 + 10 + 100 + 1000 + 10000 + 100000 + 10^6 + 10^7)$$

$$= 5555555$$

Codes

Command window

clear

clc

close all

syms x

$$Y = (5 \times 10^{-4}) * (1 + x + x.^2 + x.^3 + x.^4 + x.^5 + x.^6 + x.^7)$$

x = (0:10)

Yn = subs(Y, x)

plot(x, Yn)

xlabel('STRUCTURAL ELEMENT (M)')

ylabel('DEFORMATION')

grid on

grid minor

axis tight