

Nareel Etak Graduate Etims  
 course 2019 381  
 matric No 117/ENG02/011  
 Assignment III

1.) Solution

$$x(x-1)y'' + (3x+1)y' + y = 0$$

For the nth term

$$x(x-1)y_{n+2} + (3x+1)y_{n+1} + y_n = 0$$

$$x(x-1)y_{n+2} + y_{n+1}(2x+1) + y_n(n^2+3n+1) = 0$$

$$(y_{n+1})(n-1) + (y_n)(n^2+3n+1) = 0$$

$$(n+1)(y_{n+1}) = (n^2+3n+1)(y_n)$$

$$(n+1)(y_{n+1})_0 = (n^2+3n+1)(y_n)_0$$

$$\text{at } n=0$$

$$(y_{1+1})_0 = (0+1)(y_0)_0$$

$$(y_1)_0 = (y_0)_0$$

$$\text{at } n=1$$

$$(y_{2+1})_0 = (1+1)(y_1)_0$$

$$(y_2)_0 = 2(y_1)_0$$

$$\text{at } n=2$$

$$(y_{3+1})_0 = (2+1)(y_2)_0$$

$$(y_3)_0 = 3(y_2)_0$$

$$(y_3)_0 = 6(y_1)_0$$

$$\text{at } n=3$$

$$(y_{4+1})_0 = (3+1)(y_3)_0$$

$$(y_4)_0 = 4(y_3)_0 = 4 \times 6(y_1)_0 = 24(y_1)_0$$

$$\text{at } n=4$$

$$(y_{5+1})_0 = (4+1)(y_4)_0$$

$$3 \times 24(y_1)_0$$

$$(y_5)_0 = 120(y_1)_0$$

$$\text{at } n=5$$



$$\begin{aligned}
 (y^5)_0 &= (5+1)(y^5)_0 \\
 &= 6(y^5)_0 \\
 &= 6 \times 120 (y^5)_0 \\
 (y^6)_0 &= 720 (y^5)_0 \\
 \text{at } n=6 \\
 (y^7)_0 &= (6+1)(y^6)_0 \\
 &= 7(y^6)_0 \\
 &= 7 \times 720 (y^6)_0 \\
 (y^7)_0 &= 5040 (y^6)_0
 \end{aligned}$$

From Maclaurian

$$y = (y)_0 + \lambda (y)_1 + \frac{\lambda^2}{2!} (y^2)_0 + \frac{\lambda^3}{3!} (y^3)_0 + \frac{\lambda^4}{4!} (y^4)_0 + \dots$$

$$\begin{aligned}
 \therefore y &= (y)_0 + \lambda (y)_1 + \frac{\lambda^2}{2!} (2 (y^2)_0) + \frac{\lambda^3}{3!} (6 (y^3)_0) + \frac{\lambda^4}{4!} (24 (y^4)_0) \\
 &+ \frac{\lambda^5}{5!} (120 (y^5)_0) + \frac{\lambda^6}{6!} (720 (y^6)_0) + \dots \\
 &= (50 + 0 (y^2)_0) + \dots
 \end{aligned}$$

$$y = (y)_0 + (y)_1 \left[ \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} + \frac{\lambda^4}{24} + \frac{\lambda^5}{120} + \frac{\lambda^6}{720} + \dots \right]$$

but  $(y)_0 = 0.0005 \text{ m}$  and  $(y)_1 = 0.0005$

$$\therefore y = 0.0005 + 0.0005 \left[ \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} + \frac{\lambda^4}{24} + \frac{\lambda^5}{120} + \frac{\lambda^6}{720} + \dots \right]$$

where  $\lambda = 80$

$$y = 0.0005 + 0.0005 [5 + 20 + 53.33 + 106.67 + 213.33 + 426.67 + \dots]$$



$$y = 0.005 + 0.005 [97655]$$

$$y = 48.828m$$

where  $x = 8m$

$$y = 0.0005 + 0.0005 [8 + 64 + 512 + 4096 + 32768 + 262144]$$

$$y = 1198.8728m$$

when  $x = 0m$ .

$$y = 0.0005 + 0.0005 (10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000)$$

$$y = 0.0005 + 0.0005 [1111110]$$

$$y = 5555.5555$$

c) Command:

clear

clc

syms x

syms y

$$x = (0:10)$$

$$y = 0.0005 + 0.0005 (x + (x \cdot 2) + (x^2) + (x^3) + (x^4) + (x^5) + (x^6) + (x^7))$$

plot(x,y)

grid on

grid minor

x lab = 'x'

y lab = 'Stratum: Deformation'