YAKUBU HEEKMAT OHUNENE
COMPUTER ENGINEERING
17/ENG02/080
ENG 381

ASSIGNMENT III

- The model for the deformation (y) of a structural element is represented by the expression given in equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.005m$ and $y'(0) = 0.0005$, applying Leibnitz theorem and Maclaurin's method.

a) obtain the power series solution of the model up to and including the term in x^7.

b) Estimate the approximate deformation when $x = 5, 8$ and $10m$ and

c) With the aid of a MATLAB mfile program, plot the response of the structural element for $0 \leq x \leq 10m$

SOLUTION

a)

$$(x^2-x)y'' + (3x-1)y' + y = 0$$

$\omega_1 = (x^2-x)y''$ $\quad$ $\omega_2 = (3x-1)y'$ $\quad$ $\omega_3 = y$

$u = y^{(2)}$ $\quad v = x^2 - x$ $\qquad$ $u = y^{(1)}$ $\qquad$ $u = y$ $\quad v = 1$

$u^n = y^{(n+2)}$ $\quad v' = 2x-1$ $\qquad$ $u^n = y^{(n+1)}$ $\qquad$ $u^n = y^n$ $\quad v' = 0$

$u^{n-1} = y^{(n+1)}$ $\quad v'' = 2$ $\qquad$ $u^{n-1} = y^{(n)}$

$u^{n-2} = y^{(n)}$ $\quad v''' = 0$

Using Leibnitz Theorem

$$u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \dots$$

$$\omega_1 + \omega_2 + \omega_3 = 0$$

$$y^{(n+2)}(x^2-x) + n y^{(n+1)}(2x-1) + \frac{n(n-1)}{2!} y^n (2) + y^{(n+1)}(3x-1) + n y^n \cdot 3 + y^n = 0$$

Assuming $x = 0$

$$n(y^{(n+1)})_0(-1) + n(n-1)(y^n)_0 + (y^{(n+1)})_0(-1) + 3n(y^n)_0 + (y^n)_0 = 0$$

$$-n(y^{n+1})_0 - (y^{(n+1)})_0 + n^2 - n(y^n)_0 + 3n(y^n)_0 + (y^n)_0 = 0$$

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2-n+3n+1) = 0$$

$$(-n-1)(y^{n+1})_0 = -(n^2+2n+1)(y^n)_0 = 0$$

$$(y^{n+1})_0 = \frac{-(n^2+2n+1)}{-(n+1)}(y^n)_0$$

$$(y^{n+1})_0 = \frac{n^2+2n+1}{n+1}(y^n)_0$$

$$(y^0)_0 = 0.005m$$

$$(y^1)_0 = 0.0005m$$

when $n=1$

$$y^{1+1} = (y^2)_0 = \frac{1^2+2(1)+1}{2} = \frac{4}{2}(y^1)_0 = 2(y^1)_0 = 2\times 0.0005$$
$$= 0.001$$

when $n=2$

$$(y^3)_0 = 3(y^2)_0 = 3\times 0.001 = 0.003$$

when $n=3$

$$(y^4)_0 = 4(y^3)_0 = 4\times 0.003 = 0.012$$

when $n=4$

$$(y^5)_0 = 5(y^4)_0 = 5\times 0.012 = 0.06$$

when $n=5$

$$(y^6)_0 = 6(y^5)_0 = 6\times 0.06 = 0.36$$

when $n=6$

$$(y^7)_0 = 7(y^6)_0 = 7\times 0.36 = 2.52$$

Maclaurin Series

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}$$
$$(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^{(7)})_0 + \dots$$

$$y = 0.0005 + 0.0005x + \frac{x^2}{2!} \cdot 0.001 + \frac{x^3}{3!} \cdot 0.003 + \frac{x^4}{4!} \cdot 0.012 +$$

$$\frac{x^5}{5!} \cdot 0.06 + \frac{x^6}{6!} \cdot 0.36 + \frac{x^7}{7!} \cdot 252 + \cdots$$

$$y = 0.0005 + 0.0005x + 0.0005x^2 + 0.0005x^3 + 0.0005x^4 +$$

$$0.0005x^5 + 0.0005x^6 + 0.0005x^7 + \cdots$$

$$y = 0.0005[1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots]$$

(b)

when $x = 5m$

$$y = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7 + \cdots]$$

$$y = 48.828m$$

when $x = 8m$

$$y = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7 + \cdots]$$

$$y = 1198.3725m$$

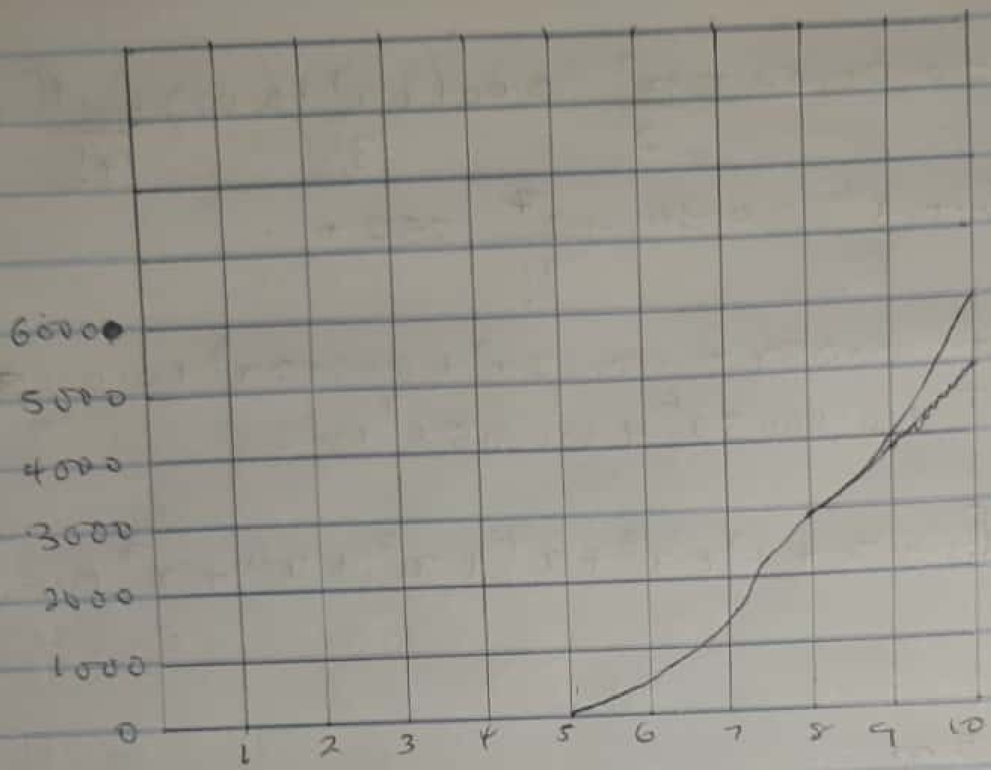
when $x = 10m$

$$y = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7 + \cdots]$$

$$y = 5555.5555m$$

(c) Command Window

```
clear
clc
close all
syms x
y = 0.0005 * [1+x+x^2+x^3+x^4+x^5+x^6+x^7]
x = 0:1:10
yn = subs(y)
ynn = double(yn)
Plot (x, ynn)
Grid on
Grid minor
```



6000
5000
4000
3000
2000
1000
0
1 2 3 4 5 6 7 8 9 10