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 Mechanical Engineering  
 17/ENG061077.

ENG 381 Assignment

1.  $x(x-1)y'' + (3x-1)y' + y = 0$

Taking  $x(x-1)y'' = u_1$   
 $(3x-1)y' = u_2$   
 $y = u_3$

considering  $u_1$

~~$y'$~~   
 $u = y''$   
 $u^n = y^{(n+2)}$   
 $u^{n-1} = y^{(n+1)}$   
 $u^{n-2} = y^n$

$v = x(x-1)$   
 $v' = 2x-1$   
 $v'' = 2$   
 $v''' = 0$

$u_2$

$u = y'$   
 $u^n = y^{(n+1)}$   
 $u^{n-1} = y^n$

$v = (3x-1)$   
 $v' = 3$   
 $v'' = 0$

$u_3$

$u = y$   
 $u^n = y^n$

$v = 1$   
 $v' = 0$

$y^n = (n!v + n \binom{n-1}{1} v' + n(n-1) \binom{n-2}{2} v'' + n(n-1)(n-2) \binom{n-3}{3} v''' + \dots$

(Recall)  
 $W_1^n = y^{n+2} \cdot (x^2-x) + n y^{(n+1)} \cdot (2x-1) + n(n-1) y^n \cdot 2 + 0$

$W_2^n = y^{(n+1)} \cdot (3x-1) + n y^n \cdot 3 + 0$

$W_3^n = y^n \cdot 1 + 0$

$\therefore W_1 + W_2 + W_3$

$$y^{n+2} = (x^2-x)ny^{n+1} \cdot (2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{(n+1)} \cdot (3x-1) +$$

$$ny^n \cdot 3 + y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2x-1)ny^{(n+1)} + (3x-1)y^{n+1} + n(n-1)y^n + 3ny^n + y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2x-n)y^{(n+1)} + (3x-1)y^{(n+1)} + (n^2-2)y^n + 3ny^n + y^n = 0$$

$$(x-1)xy^{n+2} + (2x+n+3x-n-1)y^{n+1} + (n^2+2x+1)y^n = 0$$

when  $x=0$

$$(0-1)0y^{(n+2)} + (2(0)+n+3(0)-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$-(n+1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$-(n+1)(y^{n+1})_0 + (n^2+2n+1)(y^n)_0 = 0$$

$$(y^{n+1})_0 = \frac{(n^2+2n+1)(y^n)_0}{n+1}$$

$$(y^{n+1})_0 = \frac{(n+1)(y^n)_0}{n+1}$$

$$(y^{n+1})_0 = (y^n)_0$$

when  $n=0$

$$(y^{(0+1)})_0 = (0+1)(y^0)_0$$

$$(y^1)_0 = (y^0)_0$$

when  $n=1$

$$(y^{(1+1)})_0 = (1+1)(y^{(1)})_0$$

$$(y^{(2)})_0 = 2(y^{(1)})_0$$

when  $n=2$

$$(y^{(2+1)})_0 = (2+1)(y^{(2)})_0$$

$$(y^{(3)})_0 = 3(y^{(2)})_0 = 3(2)(y^{(1)})_0$$

$$(y^{(3)})_0 = 6(y^{(1)})_0$$

when  $n=3$

$$(y^{(3+1)})_0 = (3+1)(y^{(3)})_0$$

$$(y^{(4)})_0 = 4(6)(y^{(1)})_0$$

$$(y^{(4)})_0 = 24(y^{(1)})_0$$

when  $n=4$

$$(y^{(4+1)})_0 = (4+1)(y^{(4)})_0$$

when  $n=5$

$$(y^{(6)})_0 = (5+1)(y^{(5)})_0$$

$$(y^{(6)})_0 = 6(y^{(5)})_0 = 6(120)(y^{(4)})_0$$

$$(y^{(6)})_0 = 720(y^{(4)})_0$$

when  $n=6$

$$(y^{(7)})_0 = (6+1)(y^{(6)})_0$$

$$(y^{(7)})_0 = 7(y^{(6)})_0 = 7(720)(y^{(4)})_0$$

$$(y^{(7)})_0 = 5040(y^{(4)})_0$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0$$

$$+ \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!} \cdot 2(y^{(2)})_0 + \frac{x^3}{3!} \cdot 6(y^{(3)})_0 +$$

$$\frac{x^4}{4!} \cdot 24(y^{(4)})_0 + \frac{x^5}{5!} \cdot 120(y^{(5)})_0 + \frac{x^6}{6!} \cdot 720(y^{(6)})_0 +$$

$$\frac{x^7}{7!} \cdot 5040(y^{(7)})_0$$

$$y = (1+x)(y^{(0)})_0 + (x^2+x^3+x^4+x^5+x^6+x^7)(y^{(1)})_0$$

$$y(0) = 0.0005 \text{ m}, y'(0) = 0.0005$$

$$y = (1+x)(0.0005) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$$

when  $x = 5 \text{ m}, 8 \text{ m}$  and  $10 \text{ m}$

$$y = 3 \times 10^{-3} \text{ m} + 8.825 \text{ m}$$

$$y = 8.828 \text{ m}$$

when  $x = 8 \text{ m}$

$$y = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$$

$$y = 7.5 \times 10^{-3} + 2396.736(0.0005)$$

$$y = 7.5 \times 10^{-3} + 1198$$

$$y = 1198 \text{ m}$$

when  $x = 10 \text{ m}$

$$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$$

$$y = 5.5 \times 10^{-3} + 1111100 (0.0005)$$

$$y = 5555.5556m$$

$$y = 5556m$$

Matlab.

1. Command Window

2. Clear

clc

close all

Syms x

$$x = (1+x)(0.0005) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(0.0005)$$

~~f = 0:0.01:10~~  
 $f = 0:0.01:10$

$$xt = \text{subs}(x, f)$$

$$xtn = \text{double}(xt)$$

$$\text{plot}(f, xtn)$$

$$\text{xlabel}(f)$$

$$\text{ylabel}(x)$$

$$\text{grid} \text{ on}$$

$$\text{grid} \text{ minor}$$

$$\text{axis}$$