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Electrical and Electronics

17/ENG04/032

ENG381 Assignment

Assignment:

The model for the deformation (y) of a structural element is represented by the expression given in equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005$ and $y'(0) = 0.0005$, applying Leibnitz - Mclaurin method

a) Obtain the power series solution of the model up to and including the term in k?

b) estimate the appropriate deformation when $x = 5, 8$ and 10

c) With the aid of a matlab mfile program, plot the response of the structural element for $0 \le x \le 10m$.

Solution

Using Leibnitz Method

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{\cancel{2}}\cancel{2}y^n + (3x-1)y^{n+1} + n(3)y^n + y^n = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

When $x = 0$

$$\cancel{(0^2-0)y^{n+2}} + (2n(0) - n + 3(0) - 1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$-(n+1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$y^{n+1} = \frac{(n^2+2n+1)}{(n+1)}y^n$$

$$y_0^{n+1} = \frac{(n+1)\cancel{(n+1)}}{\cancel{(n+1)}}y_0^n$$

$$y_0^{n+1} = (n+1)y_0^n$$

$n=0;\ y' = 1y^0_0$

$n=1;\ y^2_0 = 2y'_0$

$n=2;\ y^3 = 3(2)y'_0 = 6y'_0$

$n=3;\ y^4 = 4(3)(2)y' = 24(y')_0$

$n=4;\ y^5 = 5(4)(3)(2)y' = 5!\,y'_0$

$n=5;\ y^6 = 6y^5 = 6(5)(4)(3)(2)\,y'_0 = 6!\,y'_0$

$n=6;\ y^7 = 7y^6 = 7(6)(5)(4)(3)(2)\,y'_0 = 7!\,y'_0$

Mclaurin Series

$$y = y_0 + x(y'_0) + \frac{x^2}{2!}(y^2_0) + \frac{x^3}{3!}(y^3_0) + \frac{x^4}{4!}(y^4_0) + \frac{x^5}{5!}(y^5_0) + \frac{x^6}{6!}(y^6_0) + \frac{x^7}{7!}(y^7_0)$$

$$= y^0_0 + x(y'_0) + \frac{x^2}{2!}(2y'_0) + \frac{x^3}{3!}(3!\,y'_0) + \frac{x^4}{4!}(4!\,y^1_0) + \frac{x^5}{5!}(5!\,y'_0) + \frac{x^6}{6!}(6!\,y'_0) + \frac{x^7}{7}(7!\,y'_0)$$

$$= y^0_0 + x(y'_0) + x^2(y'_0) + x^3(y'_0) + x^4(y'_0) + x^5(y'_0) + x^6(y'_0) + x^7(y'_0)$$

$$= y^0_0 + y'_0(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall

$y' = y^0$

$y_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y' = 0.0005$ and $y^0 = 0.0005$, when $x = 5$

$y^5 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 0.0005(97656)$

$= 48.828$

$\simeq 49.$

when $x = 8$, $y^0_0 = 0.0005$

$y_8 = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$= 0.0005(2396745 1)$

$= 1198.3725$

$\simeq 1198$

$x = 10$

$y_{10} = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 0.0005\ (11111111)$

$= 5555.5555$

$\simeq 5556.$

ⓒ Command window

```
clc
clear all
close all
syms x,y
x = 0:0.1:10
y = (0.0005)*(1+x+x.^2+x.^3+x.^4+x.^5+x.^6+x.^7)
yn = subs(y)
ynn = double(yn)
Plot(x, ynn)
x label('x')
ylabel('T')
grid on
grid minor
axis right.
```