$x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2-x)y'' + (3x-1)y' + y = 0$ Expanding bracket

we can then assume

$(x^2-x)y'' = w_1$ and $y = w_3$

$(3x-1)y' = w_2$

Using leibnitz theorem $= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \cdots$

w_1	w_2	w_3
let $u = y''$ $v = x^2 - x$	let $u = y'$ $v = 3x-1$	let $u = y$ $v = 1$
$u^n = y^{n+2}$ $v' = 2x-1$	$u^n = y^{n+1}$ $v' = 3$	$u^n = y^n$ $v' = 0$
$u^{n-1} = y^{n+1}$ $v'' = 2$	$u^{n-1} = y^n$ $v'' = 0$	
$u^{n-2} = y^n$ $v''' = 2$		

$w_1 = (y^{n+2}\cdot(x^2-x)) + n(y^{n+1}\cdot(2x-1)) + \frac{n(n-1)y^{n-2}}{2!}$

$w_2 = (y^{n+1}\cdot(3x-1) + n y^n) + 0$

$w_3 = y^n$

$\therefore w_1 + w_2 + w_3 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + n(n-1)y^n + y^{n+1}(3x-1) + ny^n$
$+ y^n = 0$

$\therefore y^{n+2}(x^2-x) + y^{n+1}(2x-n) + y^{n+1}(3x-1) + n(n-1)y^n + ny^n\} + y^n = 0$

assuming $x = 0$

$y^{n+1}(2(0)-n) + y^{n+1}(3(0)-1) + n(n-1)y^n + ny^n\} + y = 0$

$= -ny^{n+1} - y^{n+1} + y^n(n^2-n) + 3ny^n + y^n = 0$

$= y^{n+1}(-n-1) + y^n(n^2-n+3n-1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n-1) = 0$

$y^{(n+1)}(n+1) = y^n(n^2+2n-1)$

$y^{n+1}_{(n+1)} = y^n(n+1)\cancel{(n+1)}$

$y^{n+1} = y^n(n+1)$ re-occurrence relation

$(y^{n+1})_0 = y^n(n+1)$.

$(y'')_0 = 0.0005$

$(y')_0 = 0.0005$

~~(3)~~ when $n=0$ $(y^{(0+1)})_0 = (y^{(n)})_0 (0+1)$

$(y')_0 = (y')_0 = 1$

when $n=1$ $(y^{(n+1)})_0 = (y^n)_0 (1+1)$

$(y'')_0 = (y')_0 \times 2 = 2(y')_0$

when $n=2$ $(y^{(n+1)})_0 = (y'')_0 (2+1)$

$(y''')_0 = 3(y'')_0 = 6(y')_0$

when $n=3$ $(y^{(n+1)})_0 = (y''')_0 (3+1)$

$(y^{iv})_0 = (y''')_0 4 = 24(y')_0$

when $n=4$ $(y^{v})_0 = (y^{iv})_0 5 = 120(y')_0$

when $n=5$ $(y^{vi})_0 = (y^{v})_0 6 = 720(y')_0$

when $n=6$ $(y^{vii})_0 = (y^{vi})_0 7 = 5040(y')_0$

Using maclaurin series

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{iv})_0 + \dots + \frac{x^5}{5!}120(y')_0 + \frac{x^6}{6!}720(y')_0$

$y = (y')_0 + x(y')_0 + \frac{x^2}{2!} \cdot 2(y')_0 + \frac{x^3}{3!}6(y')_0 + \frac{x^4}{4!}24(y')_0 + \frac{x^5}{5!}120(y')_0 + \frac{x^6}{6!}720(y')_0$
$+ \frac{x^7}{7!}5040(y')_0$

$y = (y')_0 (x+1) + (x^2+x^3+x^4+x^5+x^6+x^7)(y')_0$

$\therefore y = 0.0005(x+1) + (x^2+x^3+x^4+x^5+x^6+x^7)\,0.0005$

(ii) when $x=5m$

$y = 0.0005(5+1) + (5^2+5^3+5^4+5^5+5^6+5^7)\,0.0005$

$\therefore y = 48.828m$

when $x = 8m$

$y = 0.0005(8+1) + (8^2+8^3+8^4+8^5+8^6+8^7)\,0.0005$

$\therefore y = 1198.373m$

when $x=10$

$y = 0.0005(10+1) + (10^2+10^3+10^4+10^5+10^6+10^7)\,0.0005$

$\therefore y = 5555.56m$

(iii) MATLAB m-file

command window

clear

clc

close all

x = 0 : 0.1 : 10

Y = [illegible]

Yo = [illegible]

Plot (x, Yo)

[illegible]

[illegible]

[illegible]

xlabel ('x')

ylabel ('deflection')

Sketch

Graph of [illegible] against x