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Mathematics
Mechanical Engineering.

1. $y = 3e^{2x}$ and $y = 3e^{-1}$, $x = 1$, $x = 2$

for Curve $y = 3e^{x}$

$A_1 = \int_1^2 y\,dx = \int_1^2 3e^{2x}\,dx$

$A_1 = \left[\frac{3}{2} e^{2x}\right]_1^2$

$A_1 = \frac{3}{2}\left[e^{2x}\right]_1^2 = \frac{3}{2}\left[e^{2\times 2} - e^{2\times 1}\right]$

$A_1 = \frac{3}{2}\left[e^4 - e^2\right] = \frac{3}{2}\left[47.2091\right] = 70.8137$ unit^2

for Curve $y = 3e^{-x}$

$A_2 = \int_1^2 y\,dx = \int_1^2 3e^{-x}\,dx$

$= 3\int_1^2 e^{-x}\,dx = -3\left[e^{-x}\right]_1^2$

$= -3\left[e^{-2} - e^{-1}\right] = -3\left[-0.2325\right]$

$A_2 = 0.6975$ unit^2.

$\therefore$ The area bounded by the Curves $y = 3e^{2x}$ and $y = 3e^{-x}$ is $A = A_1 - A_2$

$A_1 = \left[70.8137 - 0.6975\right]$ unit^2

$A = 70.1162$ unit^2

2. $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}t$, $t = 0$ and $t = 10$

$A = \int_0^{10} y\,dx = \int_0^{10} 2\sin\frac{\pi}{10}t\,dx$

$x = 2 + 2t - 2\cos\frac{\pi}{10}t$

$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t \quad \therefore dx = \left(2 + \frac{\pi}{5}\sin\frac{\pi t}{10}\right)dt$

therefore $A = \int_0^{10}\left(2\sin\frac{\pi}{10}t\right)\left(2 + \frac{\pi}{5}\sin\frac{\pi}{10}t\right)dt$

$$= \int_0^{10} \left(4\sin\frac{\pi}{10}t + \frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt$$

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$$= \int_0^{10} \left(4\sin\frac{\pi}{10}t\right)dt + \int_0^{10}\left(\frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt$$

Integrating $\int_0^{10}\left(4\sin\frac{\pi}{10}t\right)dt$ we have;

$$\left[-\frac{40}{\pi}\cdot\cos\frac{\pi}{10}t\right]_0^{10}$$

Integrating $\int_0^{10}\left(\frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt$

from trigonometry, $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

therefore $\sin^2\frac{\pi}{10}t = \frac{1}{2}\left(1-\cos\frac{\pi}{5}t\right)$

Then Integrating we have

$$\frac{2\pi}{5}\int\left(\sin^2\frac{\pi}{10}t\right)dt$$

$$\frac{2\pi}{5}\int\left[\frac{1}{2}\left(1-\cos\frac{\pi}{5}t\right)\right]dt$$

$$\frac{2\pi}{10}\int\left(1-\cos\frac{\pi}{5}t\right)dt = \frac{2\pi}{10}\left[t-\frac{5}{\pi}\sin\frac{\pi}{5}t\right]$$

$$= \frac{\pi}{5}\left[t-\frac{5}{\pi}\sin\frac{\pi}{5}t\right]_0^{10}$$

Therefore

$$\int_0^{10}\left[4\sin\frac{\pi}{10}t + \frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right]dt =$$

$$\left[-\frac{40}{\pi}\cdot\cos\frac{\pi}{10}t + \frac{\pi}{5}t - \sin\frac{\pi}{5}t\right]_0^{10}$$

$$\left[-\frac{40}{\pi}\cdot\cos\frac{\pi}{10}t + \frac{\pi}{5}t - \sin\frac{\pi t}{5}\right]_0^{10} - \left[-\frac{40}{\pi}\cdot\frac{\cos\pi(0)}{10} + \frac{\pi}{5}(0) - \sin\frac{\pi}{5}(0)\right]_0^{10}$$

$A = 180^0$

$$\left[\frac{40}{\pi} + 2\pi - 0\right] - \left[-\frac{40}{\pi} + 0 - 0\right]$$

$$\frac{40}{\pi} + 2\pi + \frac{40}{\pi} = \left(\frac{80}{\pi} + 2\pi\right)\text{unit}^2$$

Therefore $A = 25.4648 + 6.2832$

$A = 31.748\ \text{unit}^2$