CHARLES - AMACHREE PRINCE HIBIOKPOM
18/ENG04/024
ELECT / ELECT

1) Find the area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ and the ordinates at $x = 1$ and $x = 2$

Soln

CURVE 1)

$y = 3e^{2x}$

Area $= \int y = \int_1^2 3e^{2x} \cdot dx$

$= y = \frac{3e^{2x}}{2}\Big|_1^2$

$= \frac{3e^{2(2)}}{2} - \frac{3e^{2(1)}}{2}$

$= 81.897 - 11.083$

$= 70.81$

CURVE 2

$y = 3e^{2x}$

Area $= \int y = \int 3e^{-x} \cdot dx$

$= -3e^{-x}\Big|_1^2$

$= -3e^{-2} - (-3e^{-1})$

$= -0.406 - (-1.104)$

$= 0.698$

(2) The parametric equation of a curve are $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}t$. Find the area under the curve between $t = 0$ and $t = 10$

Solution

$y = 2\sin\pi/10\, t$

$$\text{Area} = \int_{t=0}^{t=10} y = \int_{t=0}^{t=10} 2\sin\pi/10^{t} \cdot dx$$

To get dx

$$\frac{dx}{dt} = 2 + \frac{2\pi}{10}\cos\pi/10^{t}$$

$$\cancel{dx} = dt\left(2 + \frac{2\pi}{10}\sin\pi/10^{t}\right)$$

$$\int y = \int_{t=0}^{t=10}\left[\left(2\sin\pi/10^{t}\right)\left(2 + \frac{2\pi}{10}\sin\pi/10^{t}\right)\right]$$

$$\int_{t=0}^{t=10}\left[4\sin\pi/10^{t} + \frac{4\pi}{10}\sin^2\pi/10^{t}\right]$$

Let $A = \pi/10^{t}$

Recall

$\cos(A+A) = \cos A\cos A - \sin A\sin A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos^2 A = 1 - \sin^2 A$

$\cancel{\cos 2A = 1 - \sin^2}$

$\cos 2A = 1 - \sin^2 A - \sin^2 A$

$\frac{2\sin^2 A}{2} = \frac{1 - \cos 2A}{2}$

$\sin^2 A = \frac{1 - \cos 2A}{2}$

Subtitute the value into the above equation

$$4\sin \pi/10\,t + 4\pi/10 \sin^2 \pi/10\,t$$

$$4\sin \pi/10\,t + \frac{4\pi}{10}\left(\frac{1-\cos 2\pi/10}{2}\right)t$$

$$\int_{t=0}^{t=10}\left[4\sin \pi/10\,t + \frac{4\pi}{20} - \frac{4\pi}{20}\cos 2\pi/10\,t\right]$$

$$-4\left[\cos\frac{\pi}{10}t \times \frac{10}{\pi}\right]_0^{10} + \left[\frac{-4\pi \sin 2\pi/10\,t}{20} \times \frac{10}{\pi}\right]_0^{10} + \left[\frac{4\pi t}{20}\right]_0^{10}$$

$$-4\left[\cos\frac{\pi}{10}t \times \frac{10}{\pi}\right]_0^{10} + \left[\frac{4\pi t}{20} - \frac{4\pi \sin 2\pi/10\,t}{20} \times \frac{5}{\pi}\right]_0^{10}$$

$$\cancel{-4\left[\cos\frac{\pi}{10}(10) \times \frac{10}{\pi}\right]_0^{10} + \frac{4\pi}{20}\left[t - \sin 2\pi/10\,(10)\right.}$$

$$-4\left[\left(\cos\frac{\pi}{10}(10) \times \frac{10}{\pi}\right) - \left(\cos\frac{\pi}{10}(0) \times \frac{10}{\pi}\right)\right] + \frac{4\pi}{20}\left[\left(10 - \sin\frac{2\pi}{10}(10)\right) - \left(0 - \sin\frac{2\pi}{10}(0)\right)\right]$$

$$-4\left[-1 \times \frac{10}{\pi} - 1 \times \frac{10}{\pi}\right] + \frac{\pi}{5}(10-0-0-0)$$

$$-4\left[-\frac{10}{\pi} - \frac{10}{\pi}\right] + \frac{10\pi}{5}$$

$$+\frac{80}{\pi} + \frac{10\pi}{5}$$

$$25.46 + 6.283 = \underline{31.749}$$