EKPO, DEBORAH JOSEPH

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COMPUTER ENGINEERING

ENG 381 – ASSIGNMENT III

The model for the deformation (y) of a structural element is represented by the expression given in Equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin's method

a. Obtain the power series solution of the model up to and including the term in x^7.

b. Estimate the approximate deformation when $x = 5, 8$ and $10m$, and

c. With the aid of a MATLAB mfile program, plot the response of the structural element for $0 \leq x \leq 10m$.

Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Let

$B_1 = x(x-1)y''$

$B_2 = (3x-1)y'$

$B_3 = y$

B_1

$u = y^2$ $\quad v = x(2x) = x^2 - x$

$u^n = y^{(n+2)}$ $\quad v' = 2x - 1$

$u^{(n-1)} = y^{(n+1)}$ $\quad v'' = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v''' = 0$

B_2

$u = y'$ $\quad v = 3x - 1$

$u^n = y^{(n+1)}$ $\quad v' = 3$

$u^{(n-1)} = y^n$ $\quad v'' = 0$

B_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

Recall;

~~$y^n = U^nV + nU^{(n-}$~~ $y^n = U^nV + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V^2 + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V^3 + \cdots$

$$y = y^{(n+2)} \cdot (x^2 - x) + n(y^{(n-1)})(2x-1) + \frac{n(n-1)}{2!} y^{n} \cdot 2 + \frac{n(n-1)(n-2)}{3!} y^{(n-2)} \cdot 0 + (y^{n+2})(3x-1)$$

$$+ ny^{n} \cdot 3 + \frac{n(n-1)}{2!}(y^{n-1}) \cdot 0 + y'''(1) + ny^{(n-1)} = 0$$

$$= (x^2 - x)(y^{(n+2)}) + n(2x-1)(y^{(n+1)}) + n(n-1)(y^{(n)}) + 3x-1)(y^{(n+1)} + n(3)(y^{n}) \cdot (y^{n})_0$$

$$\cancel{(x^2 - x)(y^{(n+2)}) + n(2x-1)}$$

$$(x^2 - x)(y^{(n+2)}) + y^{(n+1)} \cdot + (n(2x-1) + (3x+1)) + (y^{(n)})_0 (n(n-1) + 3n + 1)$$

$$= (x^2 - x) \cdot (y^{(n+2)})_0 + (y^{(n+1)})_0 ($$

At $x = 0$

$$(y^{(n+1)})_0 ((-n-1) + (y^{n}))(n^2 + 2n + 1) = 0$$

$$(y^{(n+1)})(n+1) + (y^{(n)})_0 (n^2 + 2n + 1) = 0$$

$$(y^{(n+1)}) - (n+1) = (y^{n})_0 (n^2 + 2n + 1)$$

$$(y^{(n+1)})_0 = (y^{n}) = \frac{(n+1)(n+1)}{n+1}$$

$$(y^{(n+1)})_0 = (y^{n})_0 (n+1)$$

Recall;

$$(y^{1})_0 = 0.0005$$

$$(y')_0 = 0.0005$$

$$n = 0$$

$$(y^{(0+1)})_0 = (y^{0})(0+1)$$

$$(y^{(1)})_0 = 1(y^{0})_0$$

$$n = 1$$

$$(y^{(1+1)})_0 = (y')(1+1)$$

$$(y^{2})_0 - 2(y') = 0$$

$$n = 2$$

$$(y^{(2+1)})_0 = (y^{2})(2+1)$$

$$(y^{3})_0 = 3(y^{2}) = 3 \times 2(y') = 6y'$$

$n = 3$

$(y^{(3+1)})_0 = (y^3)(3+1)$

$(y^4)_0 = 4(y^3) = 4 \times 6(y') = 24(y')_0$

$n = 4$

$(y^{(4+1)})_0 = (y^4)(4+1)$

$(y^{(5)})_0 = 5(y^4) = 5 \times 24(y') = 120(y')_0$

$n = 5$

$(y^{(5+1)} = (y^5)(5+1)$

$(y^6)_0 = 6(y^5) = 6 \times 120(y') = 720(y')_0$

$n = 6$

$y^{(6+1)} = (y^6)(6+1)$

$(y^7)_0 = 7(y^6) = 7 \times 720(y')_0 = 5040(y')_0$

Using Leibnitz MacLaurin Theorem:

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}3(y)_2 + \frac{x^5}{5!}(2)(4)(y')_0 + \frac{x^6}{6!}-(3)(5)(y)_0 + \frac{x^7}{7!}-(2)(4)(6)(y')_0$$

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(2y')_0 + \frac{x^3}{3!}(6y') + \frac{x^4}{4!}(24y') + \frac{x^5}{5!}(120y') + \frac{x^6}{6!}(-120y') + \frac{x^7}{7!}(5040y)_0$$

$y = y^{(4)}_0(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)(y')$

$y = 0.0005(1+x) + 0.0005(x^2+x^3+x^4+x^5+x^6+x^7)$

At $x = 5$

$y = 0.0005(1+5) + 0.0005[5^2+5^3+5^4+5^5+5^6+5^7]$

$= 48.828\,m$

$x = 8$

$Y = 0.0005(1+8) + 0.0005[8^2+8^3+8^4+8^5+8^6+8^7)$

$= 1198.372\,m$

MATLAB

Command window

Clear

Clc

2!   3!   4!   8!

Close all

$x = 0 : 01 : 10$

$y = ((0.0005)(1+x)) + (6x + 2 + (x+3)(x+5) + (x \times 6) + x^{(n)}) 0.0005$

$yn = ?$

plot (x, yn)

tow = (m³)

Axson aim

Grid on

grd mirror