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17/ENG05/023
MECHATRONICS ENGINEERING
ENG381
ENGINEERING MATHEMATICS

[redacted]

1) $x(x-1)y'' + (3x-1)y' + y = 0$

Expanding the bracket

$(x^2-x)y'' + (3x-1)y' + y = 0$

$w_1 = (x^2 - x)y''$

$w_2 = (3x-1)y'$

$w_3 = y$

Using Leibnitz theorem

$u^n v + n u^{n-1} v^1 + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v_3 +$

for w_1

$u = y^2$ $v = x^2 - x$

$u^n = y^{n+2}$, $v^1 = 2x-1$

$u^{n-1} = y^{n+1}$ $v^2 = 2$

$u^{n-2} = y^n$ $v^3 = 0$

for w_2

$u = y^1$ $v = 3x-1$

$u^n = y^{n+1}$ $v^1 = 3$

$y^{n-1} = y^n$ $v^2 = 0$

For w_3

$u = y$ $v = 1$

$u^n = y^n$ $v^1 = 0$

$w_1 = y^{(n+2)} \cdot (x^2 - x) + n \cdot y^{(n+1)}(2x-1) + n(n-1)y^n \cdot 2$

$= (x^2 - x)y^{(n+2)} + n(2x-1)y^{(n+1)} + n(n-1)y^n$

$w_2 = y^{(n+1)} \cdot (3x-1) + 3ny^n$

$w_3 = y^n$.

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Summing all together

$(x^2-x)y^{(n+2)} + n(2x-1)y^{(n+1)} + (n^2-n)y^n + y^{(n+1)}(3x-1) + 3ny^n + y^n = 0$

assuming $x=0$

$(0-0)y^{n+2} = 0 + (2n(0)-n)y^{(n+1)} + (n^2-n)y^n + (3(0)-1)y^{(n+1)} + 3ny^n + y^n = 0$

$= -ny^{(n+1)} + (n^2-n)y^n - y^{(n+1)} + 3ny^n + y^n = 0$

Collecting like terms

$y^{(n+1)}(-n-1) + y^n(n^2-n+3n+1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n+1) = 0$

$(n+1)y^{(n+1)} = (n^2+2n+1)y^n$ $\quad [(n^2+2n+1) = (n+1)(n+2)]$

$(n+1)y^{(n+1)} = (n+1)(n+1)y^n$

Divide both sides by $(n+1)$

$y^{(n+1)} = (n+1)y^n$ – Recurrence relation

$(y^{n+1})_0 = y^n(n+1)$

$(y^0)_0 = 0.0005$

$(y')_0 = 0.0005$

When $n=0$

$[y^{(0+1)}]_0 = (0+1)(y^0)_0$

$[y^{(1)}]_0 = 1[y^0]_0$

$n=1;$ $[y^{(1+1)}]_0 = (1+1)(y')_0$

$[y^{(2)}]_0 = 2(y')_0$

$n=2;$ $(y^3)_0 = (2+1)y^2$

$= 3[y^2]_0 = 3[2(y^{(1)}]_0 = 6(y')_0$

$n=3;$ $(y^4)_0 = (3+1)y^3$

$4[y^3]_0 = 4(6(y')_0] = 24(y')_0$

$n=4;$ $(y^5)_0 = (4+1)y^4$

$5(y^4)_0 = 5(24(y')_0] = 120(y')_0$

$n=5;$ $(y^6)_0 = (5+1)y^5$

$= 6(y^5)_0 = 6(120(y')_0] = 720(y')_0$

$n=6$; $(Y^7)_0 = (6+1)y^6$

$= 7y^6 = 7(720(Y')_0) = 5040(Y')$

Using maclaurin series

$$Y = (Y^0)_0 + x(Y')_0 + \frac{x^2}{2!}(Y^2)_0 + \frac{x^3}{3!}(Y^3) + \frac{x^4}{4!}(Y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(Y^6)_0 + \frac{x^7}{7!}(y^7)_0 .$$

~~(Y^5)~~

$$Y = (y^0)_0 + x(y^0) + \frac{x^2}{2!}(2Y')_0 + \frac{x^3}{3!}(6Y')_0 + \frac{x^4}{4}(24y')_0 + \frac{x^5}{5!}(120y') + \frac{x^6}{6!}(720y')_0 + \frac{x^7}{7}(5040Y')$$

$$Y = Y^0(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)Y'$$

$$Y = 0.0005(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)0.0005$$

ii) Estimate the approximate deformation when $x = 5$, 8 and 10m

When $x = 5m$

$$Y = Y^0(1+5) + (5^2+5^3+5^4+5^5+5^6+5^7)0.0005.$$

$$= 0.0005(6) + (25+125+625+3125+15625+78125)$$

$$0.0005$$

$$Y = 48.828m$$

When $x = 8m$

$$Y = Y^0(1+8) + (8^2+8^3+8^4+8^5+8^6+8^7)y'$$

$$Y = 0.0005(1+8) + (64+512+4096+32768+26$$

$$2097152)0.0005$$

$$= 1198.3725m$$

when $x = 10m$

$$Y = Y^0(1+10) + (10^2+10^3+10^4+10^5+10^6+10^7)y'$$

$$= 0.0005(11) + (100+1000+10000+100\,000+1000\,000+$$

$$10\,000\,000)0.0005$$

$$= 5555.56m.$$

Matlab mfile
Command window
Clear
Clc
Close all
x = 0 : 0.01 : 10
Y = (0.0005*(1+x)) + ((x.^2+x.^3+ x.^4+x.^5 + x.^6+x.^7)*
0.005)
Yn = subs (Y)
plot (x, Yn)
x label (x, Yn)
x label ('m')
Y label ('Deflection')
axis tight
grind on
grind minor.