Nwanze Henry Chigozie
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Mechanical engineering

1) $y = 3e^{2x}$ and $y = 3e^{-1}$, $x = 1$, $x = 2$

For Curve $y = 3e^{2x}$

$$A_1 = \int_1^2 y\,dx = \int_1^2 3e^{2x}\,dx$$

$$A_1 = \left[\frac{3}{2}e^{2x}\right]_1^2$$

$$A_1 = 3/2\left[e^{2x}\right]_1^2 = \frac{3}{2}\left[e^{2\times2} - e^{2\times1}\right]$$

$$= 3/2\left[e^4 - e^2\right] = 3/2\,(47.2091)$$

$$A_1 = 70.8137 \text{ unit}^2$$

for Curve $y = 3e^{-1}$

$$A_2 = \int_1^2 y\,dx = \int_1^2 3e^{-x}\,dx$$

$$= 3\int_1^2 e^{-x}\,dx = -3\left[e^{-x}\right]_1^2$$

$$= -3\left[e^{-2} - e^{-1}\right] =$$

$$-3\left[-0.2325\right]$$

$$A_2 = 0.6975 \text{ unit}^2$$

$\therefore$ The area bounded by the Curve $y = 3e^{2x}$ and $y = 3e^{-x}$ is

$$A = A_1 - A_2$$

$$A = \left[70.8137 - 0.6975\right] \text{ unit}^2$$

$$A = 70.1162 \text{ unit}^2$$

2) $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}t$, $t = 0$ and $t = 10$

$$A = \int_0^{10} y\,dx = \int_0^{10} 2\sin\frac{\pi}{10}t\,dx$$

$$x = 2 + 2t - 2\cos\pi/10\,t$$

$$dx/dt = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t$$

$$\therefore dx = \left(2 + \frac{\pi}{5}\sin\frac{\pi t}{10}\right)dt$$

therefore $A = \int_0^{10}\left(2\sin\frac{\pi}{10}t\right)\left(2 + \frac{\pi}{5}\sin\frac{\pi}{10}t\right)dt$

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$$= \int_0^{10} \left(4 \sin \frac{\pi}{10} t\right) dt + \int_0^{10} \left(\frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt$$

Integrating $\int_0^{10} \left(4 \sin \frac{\pi}{10} t\right) dt$ we have

$$\left[-\frac{40}{\pi} \cos \pi/10\, t\right]_0^{10}$$

Integrating $\int_0^{10} \left(\frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt$.

from trogonometry; $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

therefore $\sin^2 \frac{\pi}{10} t = \frac{1}{2}(1 - \cos 2x)$

therefore $\sin^2 \frac{\pi}{10} t = \frac{1}{2} \cdot \left(1 - \cos \frac{\pi}{5} t\right)$

then Integrating we have

$$\frac{2\pi}{5} \int \left(\sin^2 \frac{\pi}{10} t\right) dt$$

$$\frac{2\pi}{5} \int \left(\frac{1}{2} \cdot \left(1 - \cos \frac{\pi}{5} t\right)\right) dt$$

$$\frac{2\pi}{10} \int \left(1 - \cos \frac{\pi}{5} t\right) dt$$

$$= \frac{\pi}{5} \left[t - \frac{5}{\pi} \sin \frac{\pi}{5} t\right]_0^{10}$$

therefore.

$$\int_0^{10} \left(4 \sin \frac{\pi}{10} t + \frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt =$$

$$\left[-\frac{40}{\pi} \cdot \cos \frac{\pi}{10} t + \frac{\pi}{5} t - \sin \frac{\pi}{5} t\right]_0^{10}$$

$$\left[-\frac{40}{\pi} \cdot \cos \frac{\pi}{10} t + \frac{\pi}{5} t - \sin \frac{\pi}{5} t\right]_0^{10} =$$

$$\left[-\frac{40}{\pi} \cdot \cos \frac{\pi}{10} (0) + \frac{\pi}{5} (0) - \sin \frac{\pi}{5} (0)\right]_0^{10}$$

$A = 180^\circ$ $A = 25.4648 + 6.2832$

Therefore $A = 31.748$ unit^2

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