NAME : JOHN-UGWU NNEOMA FAVOUR
DEPARTMENT : COMPUTER ENGINEERING
MATRIC NO : 18/ENG02/052

1. $y_1 = 3e^{2x}$ $\quad y_2 = 3e^{-x}$

$x = 1$ and $x = 2$

$$A = \int_a^b y_1 dx - \int_a^b y_2 dx$$

$$A = \int_1^2 (3e^{2x})dx - \int_1^2 (3e^{-x})dx$$

$$= \left[\left(\frac{3e^{2x}}{2x}\Big|_1^2\right) - \left(-3^{-x}\Big|_1^2\right)\right]$$

$$= \left(\frac{3e^{2(2)}}{2} - \frac{3e^{2(1)}}{2}\right) - \left(-3e^{-(2)} - (-3e^{-(1)})\right)$$

$$= (81.8972 - 11.08358) - (-0.4059 + 1.1036)$$

$$= 70.81362 - 0.6977 = 70.11592$$

$$= 70.1439 \text{ units}$$

2. $y = \frac{2\sin\pi}{10t}$ $\quad x = 2 + 2t - 2\frac{\cos\pi}{10t}$ $\quad t = 0$ and $t = 10$

$$A = \int_a^b (y)\,dx$$

$$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10t}$$

$$dx = 2 + \frac{\pi}{5}\sin\frac{\pi}{10t} \cdot dt$$

$$A = \int_0^{10}\left(2\sin\frac{\pi}{10t}\right)\left(2 + \frac{\pi}{5}\sin\frac{\pi}{10t}\right)dt$$

$$= \int_0^{10}\left(4\sin\frac{\pi}{10t} + \frac{2\pi}{5}\sin^2\frac{\pi}{10t}\right)dt$$

$$= \int_0^{10}\left(4\sin\frac{\pi}{10t}\right)dt + \frac{2\pi}{5}\int_0^{10}\left(\sin^2\frac{\pi}{10}t\right)dt$$

Recall $\sin^2 a = 1 - \cos^2 a$, where $\cos 2a = \cos^2 a - \sin^2 a$

$$\sin \frac{2\pi}{10t} = 1 - \cos^2 \left(\frac{\pi}{10}\right)^t \quad \cdots\cdots (1)$$

$$\cos \frac{\pi}{5t} = \cos^2 \frac{\pi}{10t} - \sin^2 \frac{\pi}{10t} \quad \cdots (2)$$

$$\cos^2 \frac{\pi}{10t} = 1 - \sin^2 \frac{\pi}{10t} \quad \cdots\cdots (3)$$

Substituting eqn (3) into eqn (2)

$$\cos \frac{\pi}{5}t = 1 - \sin^2 \frac{\pi}{10}t - \sin^2 \frac{\pi}{10}t$$

$$\cos \frac{\pi}{5}t = 1 - 2\sin^2 \frac{\pi}{10}t \quad \sin^2 \frac{\pi}{10}t = \frac{1 - \cos \pi/5\, t}{2}$$

$$\int_0^{10} (4 \sin \tfrac{\pi}{10}t)\,dt + \frac{2\pi}{5} \int_0^{10} \left(\frac{1 - \cos \pi/5\, t}{2}\right) dt$$

$$\int_0^{10} (4 \sin \tfrac{\pi}{10}t)\,dt + \frac{2\pi}{5} \int_0^{10} \left(1 - \cos \tfrac{\pi}{5}t\right) dt$$

$$\left(\frac{-40}{\pi} \cos \frac{\pi}{10}t \,\Big|_0^{10}\right) + \left(\frac{2\pi}{10}t - 5\,\frac{\sin \pi/5\, t}{\pi}\right)\Big|_0^{10}$$

$$\left(\frac{40}{\pi} \cos \frac{\pi}{10}t \,\Big|_0^{10}\right) + \left(\frac{\pi}{5}t - \frac{10\pi \sin \pi/5\, t}{10\pi}\right)\Big|_0^{10}$$

$$\left(-\frac{40}{\pi} \cos \frac{\pi}{5}(10) - \left(\frac{-40}{\pi} \cos \frac{\pi}{10}(0)\right)\right) + \left(\frac{\pi}{5}(10) - \sin \frac{\pi}{5}(10) - \frac{\pi}{5}(10) - \sin \frac{\pi}{5}(40)\right)$$

$$= \left(\frac{-40}{\pi} \cos \pi + \frac{40}{\pi}\right) + (2\pi - \sin 2\pi)$$

$$= \left(\frac{40}{\pi} + \frac{40}{\pi}\right) + 2\pi$$

$$= \frac{80}{\pi} + 2\pi$$

$$= 31.75 \text{ sq units}$$