Azuka Onyinye Anastasia.
Computer Engineering.
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① $y_1 = 3e^{2x}$, $y_2 = 3e^{-x}$ $x=1$ and x_2

$A = \int_a^b y_1\,dx - \int_a^b y_2\,dx$

$A = \int_1^2 (3e^{2x})\,dx - \int_1^2 (3e^{x})\,dx$

$= \left[\frac{3e^{2x}}{2}\right]_1^2 - \left[-3e^{-x}\right]_1^2$

$= \left[\frac{3e^{2(2)}}{2} - \frac{3e^{2(1)}}{2}\right] - \left[-3e^{-(2)} - 3e^{-(1)}\right]$

$= (81.8972 - 11.08358) - (-0.4059 + 1.1036$

$= 70.81362 - 0.6977$

$= 70.11592 =$

$= 70.139$ units

② $y = 2\sin \pi/10 t$, $x = 2 + 2t - 2\cos x/10 t$

$t=0$ and $t=10$

$A = \int_a^b (y)\,dx$

$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t$

$dx = 2t + \pi/5 \sin \pi/10 t\,dt$

$A = \int_0^{10} 2\sin \pi/10 t \; 2t\pi/5 \sin t / \pi/10 t)\,dt$

$= \int_0^{10} t\sin \pi/10 t + 2x/5 \sin^2 \pi/10 t)\,dt$

$= \int_0^{10} (t\sin \pi/10 t)\,dt + 2\pi/5 \int_0^{10} \sin^2 \pi/10 t)\,dt$

Recall, $\sin^2 a = 1 - \cos^2 a$, $\cos 2a = \cos^2 a - \sin^2 a$

$\sin^2 \pi/10 t = 1 - \cos^2 (\pi/10)\,t$ — (i).

$\cos \pi/5 t = \cos^2 \pi/10 t - \sin^2 \pi/10 t$ — (ii)

$\cos^2 \pi/10 t = 1 - \sin^2 \pi/10 t$ — (iii)

Substitute eqn (iii) into eqn (ii).

$\cos \pi/5 t = 1 - \sin^2 \pi/10 t - \sin^2 \pi/10 t$.

$\cos \pi/5 t = 1 - 2\sin^2 \pi/10 - \sin^2 \pi/10 t$.

$$\sin^2 \tfrac{\pi}{10}t = \frac{1-\cos\frac{\pi}{5}t}{2}$$

$$\therefore = \int_0^{10}\left(4\sin\tfrac{\pi}{10}t\right)dt + \tfrac{2\pi}{5}\int_0^{10}\left(\frac{1-\cos\frac{\pi}{5}t}{2}\right)dt$$

$$= \int_0^{10}\left(4\sin\tfrac{\pi}{10}t\right)dt + \tfrac{2\pi}{10}\int_0^{10}\left(1-\cos\tfrac{\pi}{5}t\right)dt$$

$$= \left(-\tfrac{40}{\pi}\cos\tfrac{\pi}{10}t\Big|_0^{10}\right) + \left(\frac{2\pi}{10}t - \frac{5\sin\frac{\pi}{5}t}{\pi}\right)_0^{10}$$

$$= \left(-\tfrac{40}{\pi}\cos\tfrac{\pi}{10}t\Big|_0^{10}\right) + \left(\frac{\pi t}{5} - \frac{10\pi\sin\frac{\pi}{5}t}{10\pi}\right)_0^{10}$$

$$= \left(\tfrac{-40}{\pi}\cos\tfrac{\pi}{10}(10) - \tfrac{40}{\pi}\tfrac{\cos\pi}{10}(0)\right) + \left(\tfrac{\pi}{5}(10) - \sin\tfrac{\pi}{5}(10) - \tfrac{\pi}{5}(0) - \sin\tfrac{\pi}{5}(0)\right)$$

$$= \left(-\tfrac{40}{\pi}\cos\pi + \tfrac{40}{\pi} + 2\pi - \sin 2\pi\right)$$

$$= \left(\frac{40}{\pi} + \frac{40}{\pi}\right) + 2\pi$$

$$= \frac{80}{\pi} + 2\pi$$

$$= 25.4479 + 6.28319$$

$$= 31.748$$

$$= 31.75 \text{ squnits.}$$