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ENG 381

Question 4

① $\underbrace{x(x-1)}_{W_1}y'' + \underbrace{(3x-1)}_{W_2}y' + \underbrace{y}_{W_3} = 0$

$u^n = y^{n+2}$

$u^n = y^{n+2}$ $V = x(x-1)$

$u^n = y^{n+2}$ $V = x^2 - x$

$u^{n-1} = y^{n+1}$ $V' = 2x-1$

$u^{n-2} = y^n$ $V'' = 2$

$\} W_1$

$u^n = y^{n+1}$ $V = 3x-1$

$u^{n-1} = y^n$ $V' = 3$

$\} W_2$

$u^n = y^n$ $V = 1$

$u^{n-1} = 0$ $V' = 0$

$\} W_3$

$$u^n V + nu^{n-1}V' + \frac{n(n-1)u^{n-2}V^2}{2!} + \frac{n(n-1)(n-2)u^{n-3}V^3}{3!}$$

$$= y^{n+2}(x^2-x) + n(y^{n+1})(2x-1) + \frac{n(n-1)y^n 2}{2!} + (y^{n+1})(3x-1) + ny^n 3 + y^n = 0$$

$$= y^{n+2}(x^2-x) + n(y^{n+1})(2x-1) + n(n-1)y^n + (y^{n+1})(3x-1) + ny^n 3 + y^n =$$

$$y^{n+2}(x^2-x) + y^{n+1}(2nx-n+3x-1) + y^n(3n+n^2-n+1) = 0$$

$$y^{n+2}(x^2-x) + y^{n+1}(2nx-n+3x-1) + y^n(2n+n^2-1) = 0$$

When $x=0$

$y^{n+2}(0^2-0) + y^{n+1}(3(0)-1+2(0)n-n) + y^n(2n+1+n^2)=0$

$y^{n+1}(-1-n) + y^n(2n+1+n^2) = 0$

$-y^{n+1}(n+1) + y^n(2n+1+n^2) = 0$

$y^{n+1}(n+1) = y^n(2n+1+n^2) = 0$

$y^{n+1}(n+1) = y^n(n+1)(n+1)$

$y^{n+1} = \frac{y^n(n+1)(n+1)}{n+1}$

$y^{n+1} = y^n(n+1)$

when $n=0$

$(y^{n+1})|_0 = y^n(n+1)|_0$

$y^1 = y^0$

when $n=1$

$y^{(2)} = 2y^{(1)} = 2y^{(0)}$

when $n=2$

$y^{(3)} = 3y^{(2)} = 3(2)y^{(0)} = 6y^{(0)}$

when $n=3$

$y^{(4)} = 4y^3 = 24y^{(0)}$

when $n=4$

$y^{(5)} = 5y^{(4)} = 120y^{(0)}$

$n=5$

$y^{(6)} = 6y^5 = 720y^{(0)}$

$n = 6$

$y^{(7)} = 7y^6 = 7 \times 720(y^0) = 5040y^{(0)}$.

when

$y = 0.0005m \quad y^1 = 0.0005$

at $x = 0$

$y''(x^2 - x) + y^1(3x - 1) + y = 0$

$y''(0^2 - 0) + 0.0005(3(0) - 1) + 0.0005m = 0$

$0 + 0.0005(0 - 1) + 0.0005m = 0$

$-0.0005 + 0.0005m = 0$

$$m = \frac{0.0005}{0.0005}$$

$m = 1$

$y = 0.0005(1) = 0.0005$

$y^0 = 0.0005$

$y^1 = 0.0005$

$y^4 = 2y^{(1)} = 2y^0 = 2 \times 0.0005 = 0.001$

$y^3 = 3y^{(2)} = 3 \times 0.001 = 0.003$

$y^4 = 4 \times 0.003 = 0.012$

$y^5 = 5y^4 = 5 \times 0.012 = 0.06$

$y^6 = 6 \times y^5 = 6 \times 0.06 = 0.36$

$y^7 = 7 \times y^1 = 7 \times 0.36 = 2.52$

using maclaurins series

$$y = y(0) + xy'(0) + \frac{x^2}{2!}y^2(0) + \frac{x^3}{3!}y^3(0) + \frac{x^4}{4!}y^4(0)$$

$$\therefore y = 0.0005 + 0.0005(x) + \frac{0.001(x^2)}{2!} + \frac{0.003x^3}{3!} + \frac{0.012(x^4)}{4} + \frac{0.06(x^5)}{5!} + \frac{0.36(x^6)}{6!} + \frac{2.52x^7}{7!}$$

$$y = 0.0005[x^0 + x^1 + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b) Approximate deformation

when $x = 5$

$$y = 0.0005 + 0.0005(5) + \frac{0.001(5)^2}{2!} + \frac{0.003(5)^2}{3!} + \frac{0.012(5)^4}{4!} + \frac{0.06(5)^5}{5!} + \frac{0.36(5)^6}{6!} + \frac{2.52(5)^7}{7!}$$

$$Y = 48.828 \simeq 48.83$$

when $x = 8$

$$Y = 0.0005 + 0.0005(8) + \frac{0.001(8)^2}{2!} + \frac{0.003(8)^2}{3!} + \frac{0.012(8)^4}{4!} + \frac{0.06(8)^5}{5!} + \frac{0.36(8)^6}{6!} + \frac{2.52(8)^7}{7!}$$

$$Y = 0.0005 + 0.004 + 0.032 + 0.256 + 2.045 + 16.384 + 131.072 + 1048.57 = 1198.37$$

at $x = 10$

$$y = 0.0005 + 0.0005(10) + \frac{0.001(10)^2}{2!} + \frac{0.003(10)^3}{3!} + \frac{0.012(10)^4}{4!} + \frac{0.06(10)^5}{5!} + \frac{0.36(10)^6}{6!} + \frac{2.52(10)^7}{7!}$$

$$y = 0.0005 + 0.005 + 0.05 + 0.5 + 5 + 50 + 500 + 5000$$

$$y = 5555.56$$

When $x = 5$ $y = 48.83$

$x = 8$ $y = 1198.37$

$x = 10$ $y = 5555.56$

MATLAB CODE

```
Command window
clear
clc
Close all
Syms x, y
x = 0:1:10
y = 0.0005*[(x^0) + (x^1) + (x^2) + (x^3) + (x^4) + (x^5) + (x^6) + (x^7)]
plot (x, y)
xlabel = ('x')
ylabel = ('y')
```