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1. $y_1 = 3e^{2x}$, $y_2 = 3e^{-x}$.

$x = 1$ and $x = 2$.

$$A = \int_a^b y_1 \, dx - \int_a^b y_2 \, dx.$$

$$A = \int_1^2 (3e^{2x})dx - \int_1^2 (3e^{-x})dx.$$

$$= \left(\frac{3e^{2x}}{2x}\Big|_1^2 - \left(-3^{-x}\Big|_1^2\right)\right)$$

$$= \left(\frac{3e^{2(2)}}{2} - \frac{3e^{2(1)}}{2}\right) - \left(-3e^{-(2)} - (-3e^{-(1)})\right)$$

$$= (81.8972 - 11.08358) - (-0.4059 + 1.1036).$$

$$= 70.81362 - 0.6977.$$

$$= 70.11592 = 70.1439 \text{ units}$$

2. $y = \frac{2\sin\pi}{10t}$ $x = 2 + 2t - 2\frac{\cos\pi}{10t}$

$t = 0$ and $t = 10$.

$$A = \int_a^b (y)\, dx.$$

$$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10t}$$

$$dx = 2 + \frac{\pi}{5}\sin\frac{\pi}{10t}\, dt$$

$$A = \int_0^{10} \left(2\sin\frac{\pi}{10t}\right)\left(2 + \frac{\pi}{5}\sin\frac{\pi}{10t}\right)dt$$

$$= \int_0^{10} \left(4\sin\frac{\pi}{10t} + \frac{2\pi}{5}\sin^2\frac{\pi}{10t}\right)dt$$

$$= \int_0^{10} \left(4 \sin \frac{\pi}{10t}\right) dt + \frac{2\pi}{5} \int_0^{10} \left(\sin^2 \frac{\pi}{10} t\right) dt$$

Recall $\sin^2 a = 1 - \cos^2 a$

$\cos 2a = \cos^2 a - \sin^2 a$

$\sin \frac{2\pi}{10t} = 1 - \cos^2 (\pi/10)t$ ----①

$\cos \frac{\pi}{5} t = \cos^2 \frac{\pi}{10} t - \sin^2 \frac{\pi}{10} t$ ----②

$\therefore \cos^2 \frac{\pi}{10} t = 1 - \sin^2 \frac{\pi}{10} t$ ------③

Substituting eqn ③ into eqn ②

$\cos \frac{\pi}{5} t = 1 - \sin^2 \frac{\pi}{10} t - \sin^2 \pi/10 t$

$\cos \frac{\pi}{5} t = 1 - 2 \sin^2 \frac{\pi}{10} t \quad \sin^2 \frac{\pi}{10} t = \frac{1 - \cos \frac{\pi}{5} t}{2}$

$$= \int_0^{10} (4 \sin \pi/10 t) dt + \frac{2\pi}{5} \int_0^{10} \left(\frac{1 - \cos \pi/5 t}{2}\right) dt$$

$$= \int_0^{10} (4 \sin \pi/10 t) dt + \frac{2\pi}{5} \int_0^{10} \left(1 - \cos \frac{\pi}{5} t\right) dt$$

$$= \left(\frac{-40}{\pi} \cos \frac{\pi}{10} t \Big|_0^0\right) + \left(\frac{2\pi}{10} \left(t - \frac{5 \sin \pi/5 t}{\pi}\right)\Big|_0^{10}\right.$$

$$= \left(\frac{-40}{\pi} \cdot \cos \frac{\pi}{10} t \Big|_0^{10}\right) + \left(\frac{\pi}{5} t - \frac{10\pi \sin \pi/5 t}{10\pi}\right)\Big|_0^{10}$$

$$\left(\left(\frac{-40}{\pi} \cos \frac{\pi}{5} (10) - \left(\frac{-40}{\pi} \cos \pi/10 (0)\right)\right) + \left(\left(\frac{\pi}{5} (10) - \sin \frac{\pi}{5} (10) - \right.\right.\right.$$

$$\left.\left.\left.\frac{\pi}{5} (10) - \sin \frac{\pi}{5} (0)\right]\right.\right.$$

$$= \left(-\frac{40}{\pi} \cos \pi + \frac{40}{\pi}\right) + (2\pi - \sin 2\pi)$$

$$= \left(\frac{40}{\pi} + \frac{40}{\pi}\right) + 2\pi$$

$$= \frac{80}{\pi} + 2\pi$$

$= 31.75$ sq units