OSAROEKIYE PRECIOUS ONYIA 18/ENG01/020
CHEMICAL ENGINEERING

① Find the area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ and the ordinates at $x = 1$ and $x = 2$

Soln

Curve 1

$y = 3e^{2x}$

$\text{Area} = \int y = \int_1^2 3e^{2x} \cdot dx$

$= y = \frac{3e^{2x}}{2}\Big|_1^2$

$= \frac{3e^{2(2)}}{2} - \frac{3e^{2(1)}}{2}$

$= 81.897 - 11.083$

$= \underline{\underline{70.81}}$

Curve 2

$y = 3e^{2x}$

$\text{Area} = \int y = \int 3e^{-x} \cdot dx$

$= -3e^{-x}\Big|_1^2$

$= -3e^{-2} - (-3e^{-1})$

$= -0.406 - (-1.104)$

$= \underline{\underline{0.698}}$

(2) The parametric equation of a curve are $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}$. Find the area under the curve between $t = 0$ and $t = 10$

Soln

$y = 2\sin\frac{\pi}{10}t$

$\text{Area} = \int_{t=0}^{t=10} y = \int_{t=0}^{t=0} 2\sin\frac{\pi}{10}t \cdot dx$

$\frac{dx}{dt} = 2 + \frac{2\pi}{10}\cos\frac{\pi}{10}t$

$dx = dt\left(2 + \frac{2\pi}{10}\sin\frac{\pi}{10}t\right)$

$\int y = \int_{t=0}^{t=10} \left[\left(2\sin\frac{\pi}{10}t\right)\left(2 + \frac{2\pi}{10}\sin\frac{\pi}{10}t\right)\right]$

$$\int_{t=0}^{t=10}\left[4\sin\frac{\pi}{10}t + \frac{4\pi}{10}\sin^2\frac{\pi}{10}t\right] \qquad \text{let } A = \frac{\pi}{10}t$$

Recall

$$\cos(A+A) = \cos A\cos A - \sin A\sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos 2A = 1 - \sin^2 A - \sin^2 A$$

$$\frac{2\sin^2 A}{2} = \frac{1-\cos 2A}{2}$$

$$\sin^2 A = \frac{1-\cos 2A}{2}$$

Substitute the value into the above equation

$$4\sin \pi/10\, t + 4\pi/10 \sin^2 \pi/10\, t$$

$$4\sin \pi/10\, t + 4\pi/10\left(\frac{1-\cos 2\pi/10}{2}\right)t$$

$$\int_{t=0}^{t=10}\left[4\sin \pi/10\, t + \frac{4\pi - 4\pi\cos \pi/10\, t}{20}\right]$$

$$-4\left[\cos\frac{\pi}{10}t \times \frac{10}{\pi}\right]_0^{10} + \left[\frac{-4\pi\sin^2 2\pi/10t}{20} \times \frac{10}{\pi}\right]_0^{10} + \left[\frac{4\pi t}{20}\right]_0^{10}$$

$$-4\left[\cos\frac{\pi}{10}t \times \frac{10}{\pi}\right]_0^{10} + \left[4\pi t - 4\pi\sin 2\pi/10t \times \frac{5}{\pi}\right]_0^{10}$$

$$-4\left[\left(\cos\frac{\pi}{10}(10) \times \frac{10}{\pi}\right) - \left(\cos\frac{\pi}{10}(0) \times \frac{10}{\pi}\right)\right] + \frac{4\pi}{20}\left[\left(10 - \sin 2\pi/10^{(10)}\right)\left(-0 - \sin 2\pi\right.\right.$$

$$-4\left[-1 \times \frac{10}{\pi} - 1 \times \frac{10}{\pi}\right] + \frac{\pi}{5}\left(10-0-0-0\right)$$

$$-4\left[\frac{-10}{\pi} - \frac{10}{\pi}\right] + \frac{10\pi}{5} = +\frac{80}{\pi} + \frac{10\pi}{5}$$

$$25.46 + 6.283 = \underline{\underline{31.749}}$$