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ENG 281

ENGINEERING MATHS

Find the area bounded by the curve $y = 3e^{2x}$ and $y = 3e^{-x}$ and the ordinate at $x = 1$ and $x = 2$

Solution

$$y = 3e^{2x}$$

$$y = 3e^{-x}$$

$$x = 1, x = 2$$

$$\text{let } y_1 = 3e^{2x}$$

$$y_2 = 3e^{-x}$$

$$\text{Using, } A = \int_a^b y_1 - y_2 \, dx$$

$$A = \int_1^2 3e^{2x} - 3e^{-x} \, dx$$

$$A = \left[\frac{3e^{2x}}{2} + 3e^{-x} \right]_1^2$$

$$A = \left(\frac{3e^{2 \cdot 2}}{2} + 3e^{-2} \right) - \left(\frac{3e^{2 \cdot 1}}{2} + 3e^{-1} \right)$$

$$A = 81.9 - 11.1$$

$$\therefore A = 70.8 \text{ Unit}^2$$

$$\text{when } y_1 = 3e^{-x}$$

$$\therefore A = \int_a^b y_1 \, dx \quad A_{\text{total}} = (A + A_{\text{below}})$$

$$A = \int_1^2 3e^{-x} \, dx = -3e^{-x} \Big|_1^2 = -3e^{-2} - (-3e^{-1})$$

$$A = (-3e^{-2}) - (-3e^{-1})$$

$$A = -0.4 - (-1.1)$$

$$A = -0.4 + 1.1$$

$$A = 0.7 \text{ units}$$

$$A = 70.8 - 0.7$$

$$A = 70.1 \text{ unit}^2$$

The Parametric equations of a curve are $y = 2 \sin \frac{\pi}{10} t$ and $x = 2 + 2t - 2 \cos \frac{\pi}{10} t$. Find the area under the curve between $t = 0$ and $t = 10$.

Solution

$$y = 2 \sin \frac{\pi}{10} t, \quad x = 2 + 2t - 2 \cos \frac{\pi}{10} t$$

$$\begin{aligned} t &= 0 & t \cdot t_0 &= t_0 = 0, \\ t &= 10 & t_0 &= 10 \end{aligned}$$

$$\frac{dx}{dt} = 2 + \frac{\pi}{5} \sin \frac{\pi}{10} t$$

Using, $A = \int_a^b y \, dx = \int_{t_0}^{t_1} y \cdot \frac{dx}{dt} \, dt$

$$A = \int_0^{10} \left(2 \sin \left(\frac{\pi}{10} t \right) \right) \cdot \left(2 + \frac{\pi}{5} \sin \frac{\pi}{10} t \right) \, dt$$

$$A = \int_0^{10} 4 \sin \left(\frac{\pi}{10} t \right) + \frac{2\pi}{5} \sin^2 \left(\frac{\pi}{10} t \right) \, dt$$

$$A = 4 \int_0^{10} \sin \left(\frac{\pi}{10} t \right) + \frac{2\pi}{5} \int_0^{10} \sin^2 \left(\frac{\pi}{10} t \right) \, dt$$

$$\text{Let } A = (\pi/10)t$$

$$\text{call that } \cos(A+A) = \cos A \cos A - \sin A \sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = (1 - \sin^2 A) - \sin^2 A$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\cos 2A - 1 = 2\sin^2 A$$

$$1 - \cos 2A = 2\sin^2 A$$

$$1 - \frac{\cos 2A}{2} = \sin^2 A$$

$$A = 4 \int_0^{10} \sin \left(\frac{\pi}{10} t \right) + \frac{2\pi}{5} \cdot \frac{1}{2} \int_0^{10} 1 - \cos^2 \left(\frac{\pi}{10} t \right) \, dt$$

$$A = \frac{-4 \cos(\pi/10 t)}{\pi/10} + \left(\frac{2\pi}{10}\right) \left(\frac{t - \sin^2 \pi/10 t}{\pi/10} \right)$$

$$A = \frac{-4 \cos(\pi/10)}{\pi/10} + \left(\frac{2\pi}{10}\right) \left[\frac{10 - \sin^2(\pi/10)}{\pi/10} \right]$$

$$= 19.01 - (-12.73) = 31.74 \text{ unit}^2$$