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Leibnitz method

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2 - x)y'' + (3x-1)y' + y = 0$$

$$(x^2 - x)y^{n+2} + (3x-1)y^{n+1} + n(n-1) \frac{2y^n}{2} + (3x-1)y^{n+1} +$$

$$y^{n+1} + n(3)y^n + y = 0$$

$$(x^2 - x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3y^n + y^n = 0$$

$$(x^2 - x)y^{n+2} + (2xn - n)y^{n+1} + (n^2 - n)y^n + (3x-1)y^{n+1} + 3y^n + y^n = 0$$

$$(x^2 - x)y^{n+2} + (2xn - n + 3x - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

When $x=0$, $(2n(0) - n + 3(0) - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$

$$(-n - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$-(n+1)y^{n+1} = -(n^2 + 2n + 1)y^n$$

$$(y^{n+1})_0 = \frac{(n^2 + 2n + 1)y^n}{(n+1)}$$

$$(y^{n+1})_0 = \frac{(n+1)(n+1)(y^n)_0}{(n+1)}$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

$n+1 = 0$; $y^1 = 1y^0$

$n = 1$; $y^2 = 2y^0$

$n = 2$; $y^3 = 3y^1 = 3(2)y^0 = 6(y^0)_0$

$n = 3$; $y^4 = 4y^2 = 4(6)(2)y^0 = 24(y^0)_0$

$n = 4$; $y^5 = 5y^3 = 5(24)(3)(2)y^0 = 5!(y^0)_0$

$$n=5; y^1 = 6y^5 = 6(5)(4)(3)(2)y^5 = 6!(y^1)_0$$

$$n=6; y^7 = 7y^6 = 7(6)(5)(4)(3)(2)y^6 = 7!(y^1)_0$$

When
= 0.0

Using Maclaurin Series

$$y = y_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0$$

$$+ \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$$

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$$y = y_0 + 0!(y^1)_0 + \frac{x^2}{2!}(2y^2)_0 + \frac{x^3}{3!}(3!y^3)_0 + \frac{x^4}{4!}(4!y^4)_0$$

$$+ \frac{x^5}{5!}(5!y^5)_0 + \frac{x^6}{6!}(6!y^6)_0 + \frac{x^7}{7!}(7!y^7)_0$$

$$= y_0 + y_1(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall $y^1 = y^0$

$$y_0 = (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

$$y_1 = 0.0005$$

$$y_0 = 0.0005, \text{ when } x = 5$$

$$y^5 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$

$$= 0.0005(77656)$$

$$= 48.828 \approx 49$$

$$\text{when } x = 8 \quad y_0 = 0.0005$$

$$y^8 = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$$

$$= 0.0005(22967457)$$

$$= 1148.3728 \approx 1148$$

$$\begin{aligned} \text{When } x = 10 \quad y_{10} &= 0.0005 (1 + 10 + 10^2 + 10^3 + 10^4 \\ &\quad + 10^5 + 10^6 + 10^7) \\ &= 0.0005 (1111111) \\ &= 5555 - 5.555 \approx 5556 \end{aligned}$$

b) Command window

clc

clear sH

close sH

syms x y

x = 0:0:1:10

y = (0.0005)^x * (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)

yn = subs(y)

yn = double(yn)

plot(x, yn)

xlabel('x')

ylabel('T')

grid on

grid minor

axis tight