

using Leibnitz method

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y'' + (3x-1)y' - y = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n \frac{(x^2-x)^n}{2} y'' + n(3x-1)y^n + y^n = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3y^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3y^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$y^n = 0$$

when $x = 0$

$$(2n(0)-n+3(0)-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$y^{n+1} = \frac{(n^2+2n+1)y^n}{n+1}$$

$$y^{n+1} = \frac{(n^2+2n+1)y^n}{(n+1)}$$

$$(y^{n+1})_0 = \frac{(n^2+2n+1)}{(n+1)} y_0$$

$$(y^{n+1})_0 = \frac{(n+1)}{(n+1)} (y^n)_0$$

$$(y^{n+1})_0 = (n+1) (y^n)_0$$

$$\begin{aligned}
 \text{At } n=0; & \quad y_1 = 1y^0 \\
 n=1; & \quad y_2 = 2(y)_0 \\
 n=2; & \quad y_3 = 3y^2 = 3(2)y' = 6(y')_0 \\
 n=3; & \quad y_4 = 4y^3 = 4(3)(2)y' = 24(y')_0 \\
 n=4 & \quad y_5 = 5y^4 = 5(4)(3)(2)y' = 5!(y')_0 \\
 n=5 & \quad y_6 = 6y^5 = 6(5)(4)(3)(2)y' = 6!(y')_0 \\
 n=6 & \quad y_7 = 7y^6 = 7(6)(5)(4)(3)(2)y' = 7!(y')_0
 \end{aligned}$$

Maclaurin Series

$$y = y_0 + x(y^2)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 +$$

$$\frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$$

$$= y^0 + x(y'_0) + \frac{x^2}{2!}(2y'_0) + \frac{x^3}{3!}(3!y'_0)$$

$$+ \frac{x^4}{4!}(4!y'_0) + \frac{x^5}{5!}(5!y'_0) + \frac{x^6}{6!}(6!y'_0) +$$

$$\frac{x^7}{7!}(7!y'_0)$$

$$= y^0 + x(y'_0) + x^2(y'_0) + x^3(y'_0) + x^4(y'_0) +$$

$$x^5(y'_0) + x^6(y'_0) + x^7(y'_0)$$

$$= y^0 + y'_0 (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

recall

$$y' = y'$$

$$y_0 = (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

$$y' = 0.0005$$

$$y^0 = 0.0005, \text{ when } x=5$$

$$\begin{aligned} y^3 &= 0.0005 (1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7) \\ &= 0.0005 (97656) \\ &= 48.828 \approx 49 \end{aligned}$$

$$\text{when } x=8, y_0 = 0.0005$$

$$\begin{aligned} y^8 &= 0.0005 (1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7) \\ &= 0.0005 (2396745) \\ &= 1198.3725 \\ &\approx 1198 \end{aligned}$$

$$x=10$$

$$\begin{aligned} y_{10} &= 0.0005 (1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 \\ &\quad + 10^7) \\ &= 0.0005 (11111111) \\ &= 5555.5555 \\ &\approx 5556 \end{aligned}$$

Command window

clc

clear all

close all

SymS x y

xc = 0:0.1:10

$$y = (0.0005) (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

yn = subs (y)

yn = double (yn)

plot (xc, yn)

x label ('x')

y label ('T')

grid on

grid minor

axis tight