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Assignment.

a. $\frac{dy}{dx} + y\tanh x = 2\sinh x$.

$P = \tanh x$ $\quad Q = 2\sinh x$.

$\int P dx = \int \tanh x \, dx$

$\int \frac{\sinh x}{\cosh x} dx$

Let $u = \cosh x$.

$\frac{du}{dx} = \sinh x$.

$dx = \frac{du}{\sinh x}$.

$\int \frac{\sinh x}{u} \frac{du}{\sinh x}$

$= \int \frac{1}{u} du = \ln u$

$= \ln \cosh x$

Using Integrating factor.

$y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot e^{\ln \cosh x} = \int 2\sinh x \cdot e^{\ln \cosh x} dx$.

$y \cdot \cosh x = 2\int \sinh x \cdot \cosh x \, dx$.

Integrating By parts.

$\int \underset{v}{\sinh x} \cdot \underset{du}{\cosh x} = uv - \int u dv$.

$= \sinh x \cdot \sinh x - \int \sinh x \cdot \cosh x$.

$\int \sinh x \cdot \cosh x + \int \sinh x \cdot \cosh x = \sinh^2 x$

$2\int \sinh x \cdot \cosh x = \sinh^2 x$

$\int \sinh x \cdot \cosh x = \frac{\sinh^2 x}{2}$.

Hence.

$y \cdot \cosh x = 2 \times \frac{\sinh^2 x}{2} \longrightarrow y \cdot \cosh x = \sinh^2 x + c$.

$y = \frac{\sinh^2 x}{\cosh x} + \frac{c}{\cosh x}$. $\longrightarrow y = \tanh x \sinh x + \frac{c}{\cosh x}$

b. $\frac{dy}{dx} + 2y = e^{3x}$.

$P = 2$ $\quad Q = e^{3x}$. ; $\int P dx = \int 2 dx = 2x$.

Using Integrating Factor

$y \cdot IF = \int Q \cdot IF dx$.

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \cdot dx$

$y \cdot e^{2x} = \int e^{5x} dx$ $\longrightarrow$ $y \cdot e^{2x} = \frac{e^{5x}}{5} + c$.

$y = \frac{e^{3x}}{5} + \frac{c}{e^{2x}}$.

c. $x\frac{dy}{dx} = x^2+2x-3.$

By seperating variables

$\cancel{dy} \; dy = \frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x} \, dx.$

$dy = x + 2 - \frac{3}{x} \, dx.$

Integrating Both sides.

$\int dy = \int x + \int 2 - \int \frac{3}{x}$

$x2x^2.$

$y = \frac{x^2}{2} + 2x + \frac{3}{x^2} + c.$

$y = x^4 + 4x^3 + 6 + 2cx^2$

d. $\frac{dy}{dx} + \frac{y}{x} = y^3$

Using Bernoulli's equation.

Dividing through by y^3

$y^{-3}\frac{dy}{dx} + \frac{y^{-2}}{x} = 1$

$z = y^{1-3}$; $z = y^{-2}.$

$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}.$

Multiply through by -2

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2.$

Substituting value of z and $\frac{dz}{dx}$

$\frac{dz}{dx} - \frac{2z}{x} = -2.$

$P = -2/x \qquad Q = -2.$

$\int P dx = \int -2/x \, dx \rightarrow -2\int 1/x = -2\ln x.$

Using Integrating Factor.

$z \cdot IF = \int Q \cdot IF$

$z \cdot e^{-2\ln x} = \int -2 \cdot e^{-2\ln x}.$

$z \cdot e^{\ln(x)^{-2}} = \int -2 \cdot e^{\ln(x)^{-2}}.$

$z \cdot x^{-2} = \int -2 \cdot x^{-2}$

$\frac{z}{x^2} = \int \frac{-2}{x^2}. \rightarrow \frac{z}{x^2} = -2\int \frac{1}{x^2}$

$\frac{z}{x^2} = -2 \times \frac{x^{-2+1}}{-2+1} \rightarrow \frac{z}{x^2} = -2 \times \frac{x^{-1}}{-1}$

$\frac{z}{x^2} = 2x^{-1} + c,$

$z = 2x^{-1+2} + x^2c,$

$z = 2x + x^2c$; Recall $z = y^{-2}$

$y^{-2} = 2x + x^2c$

$y = (2x + x^2c)^{-1/2}$

e. $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

By seperating variables.

$dy = \frac{x^3\sin 3x}{x^2} + \frac{4}{x^2} \, dx.$

$dy = x\sin 3x + \frac{4}{x^2} \, dx$

Integrating Both sides.

$y = \int x\sin 3x + \int \frac{4}{x^2} \, dx.$

Integrating By parts

$\int \underset{v}{x} \, \underset{du}{\sin 3x} = uv - \int u \, dv$

$= x \cdot \frac{-\cos 3x}{3} - \int -\frac{\cos 3x}{3} \cdot 1$

$= -\frac{1}{3}x\cos 3x + 1/3\int \cos 3x.$

$= -\frac{1}{3}x\cos 3x + \frac{1}{3}\left[\frac{\sin 3x}{3}\right]$

$= -1/3\, x\cos 3x + \frac{1}{9}\sin 3x.$

$\int \frac{4}{x^2} = \int 4x^{-2} = \frac{4x^{-2+1}}{-2+1}$

$= -4x^{-1}$

Hence

$y = -1/3\, x\cos 3x + \frac{1}{9}\sin 3x - \frac{4}{x} + c$

f. $(x^3+xy^2)\frac{dy}{dx} = 2y^3$

$$\frac{dy}{dx} = \frac{2y^3}{x^3+xy^2}.$$

Substituting $y = vx$. and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^3+x(vx)^2}.$$

$$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3+x^3v^2}.$$

$$v + x\frac{dv}{dx} = \frac{x^3\,2v^3}{x^3(1+v^2)}$$

$$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}.$$

$$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v \;;\; x\frac{dv}{dx} = \frac{2v^3 - v(1+v^2)}{1+v^2}.$$

$$x\frac{dv}{dx} = \frac{2v^3 - v - v^3}{1+v^2}.$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

By. seperating variables.

$$\frac{1+v^2}{v^3-v}\,dv = \frac{1}{x}\,dx.$$

$$\frac{1+v^2}{v(v^2-1)}\,dv = \frac{1}{x}\,dx.$$

$$\frac{1+v^2}{v(v+1)(v-1)}\,dv = \frac{1}{x}\,dx$$

Integrating both sides.

$$\int \frac{1+v^2}{v(v+1)(v-1)}\,dv = \int \frac{1}{x}\,dx.$$

Integrating by partial fraction

$$\frac{1+v^2}{v(v+1)(v-1)} = \frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}$$

$$1+v^2 = A(v+1)(v-1) + B(v-1)v + C\,v(v+1)$$

when $v = 1$

1

$$1+1^2 = A(1+1)(1-1) + B(1-1)1 + C(1)(1+1)$$

$$2 = 0+0+2C$$

$$C = 1$$

when $v = -1$

$$1+(-1)^2 = A(-1+1)(-1-1) + B(-1-1)\cdot -1 + C(-1)(-1+1)$$

$$2 = 2B + 0 + 0$$

$$B = 1$$

when $v = 2$.

$$1+2^2 = A(2+1)(2-1) + B(2-1)2 + C(2)(2+1)$$

$$5 = 3A + 2B + 6C$$

$$5 = 3A + 2(1) + 6(1)$$

$$5 = 3A + 8$$

$$3A = 5-8 \;;\; 3A = -3 \;;\; A = -1$$

$$\int \frac{-1}{v} + \int \frac{1}{v+1} + \int \frac{1}{v-1}$$

$$= -\ln v + \ln(v+1) + \ln(v-1).$$

Hence, Integrating both sides becomes.

$$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + c.$$

Let $c = \ln A$

$$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + \ln A.$$

$$\ln(v)^{-1} + \ln(v+1) + \ln(v-1) = \ln x + \ln A.$$

$$\ln v(v+1)(v-1) = \ln A\cdot x.$$

$$v(v+1)(v-1) = A\cdot x$$

Recall that $v = y/x$ $(y = vx)$.

$$y/x\,(y/x + 1)(y/x - 1) = A\cdot x.$$