ASSIGNMENT 2

(a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

(b) $\frac{\partial y}{\partial x} + 2y = e^{3x}$

Comparing with equation $\frac{dy}{dx} + P_y = Q$

$P = 2$, $Q = e^{3x}$

$\therefore \quad IF = e^{\int p\,dx}$; $\int p\,dx = \int 2\,dx$

$= 2x$

$\therefore \quad IF = e^{2x}$

Recall; $y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x} \, dx$

$y \cdot e^{2x} = \frac{e^{5x}}{5} + C$

$y = \frac{e^{5x}}{e^{2x} \, 5} + \frac{C}{e^{2x}}$

$y = \frac{e^{3x}}{5} + \frac{C}{e^{2x}}$

) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

By direct integration,

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x}$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

) $\frac{dy}{dx} + \frac{y}{x} = y^3$; Divide through by y^3

$y^{-3}\frac{dy}{dx} + \frac{1}{x} \cdot y^{-2} = 1$ — eq 1

Hence, $z = y^{-2}$ $\therefore$ $\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$

Multiply eq 1 by (-2)

$-2y^{-3}\frac{dy}{dx} - \frac{2}{x} \cdot y^{-2} = -2$ — eq 2

Since $-2y^{-3}\frac{dy}{dx} = \frac{dz}{dx}$ and $z = y^{-2}$

$\therefore$ $\frac{dz}{dx} - \frac{2}{x}z = -2$ — eq 3

Where $-\frac{2}{x} = P$, $Q = -2$

$IF = e^{\int p dx}$; $\int p dx = \int -2/x \, dx$

$= -2\ln x$

$IF = e^{-2\ln x}$

$IF = e^{\ln x^{-2}}$ $\therefore IF = x^{-2} = 1/x^2$

$z \cdot IF = \int Q \cdot IF \, dx$

Recall $z = y^{-2}$

$y^{-2} \cdot x^{-2} = \int -2 \cdot 1/x^2 \, dx$

$y^{-2} \cdot x^{-2} = -2\ln x^2 + c$

$y^{-2} = -2x^2 \ln x^2 + Cx^2$

$y^2 = \frac{1}{-2x^2 \ln x^2} + \frac{1}{Cx^2}$

(e) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$

Using direct integration

$\int \frac{dy}{dx} = \int x \sin 3x + \frac{4}{x^2}$

$y = \frac{-1x}{3} \cos 3x - \frac{1}{9} \sin 3x - \frac{4}{x} + C$

(f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$

Recall $\frac{dy}{dx} = v + x\frac{dv}{dx}$, $y = vx$

$\therefore$ $v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$

$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3 + x^3v^2}$

$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1 + v^2)}$

$$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$$

$$x\frac{dv}{dx} = \frac{2v^3 - v - v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

Seperating values

$$\frac{1+v^2}{v^3 - v}\, dv = \frac{1}{x}\, dx$$

$$\int \frac{1+v^2}{v^3 - v}\, dv = \int \frac{1}{x}\, dx$$

$$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + C$$

$$\ln v^{-1} + \ln(v+1) + \ln(v-1) = \ln x + C$$

$$\ln\left[v^{-1} \cdot (v+1) \cdot (v-1)\right] = \ln x + C$$

$$\ln\left[\frac{1}{v} \cdot (v^2 - 1^2)\right] = \ln x + C$$

$$\ln\left[v - \frac{1}{v}\right] = \ln x + C$$

Let $C = \ln A$

$$\ln\left[v - \frac{1}{v}\right] = \ln x + \ln A$$

$$\ln\left[\frac{v-1}{v}\right] = \ln Ax$$

$$\frac{v-1}{v} = Ax$$

$y = vx \therefore v = y/x$

$$\frac{y}{x} - \frac{x}{y} = Ax$$

$$\frac{y^2 - x^2}{xy} = Ax$$

$$y^2 - x^2 = x^2 Ay$$

$$y^2 = Ax^2 y + x^2$$