Kazie Nnenna
Mechatronics
18/ENG051026

1 $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$\int P dx = \ln\cosh x$

$IF = e^{\int P dx} = e^{\ln\cosh x} = \cosh x$

$y \cdot IF = \int Q \cdot IF$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x$

$y \cdot \cosh x = \frac{2\sinh^2 x}{2} + 2c$

$y \cdot \cosh x = \sinh^2 x + 2c$

$y = \frac{\sinh^2 x}{\cosh x} + \frac{2c}{\cosh x}$

$y = \sinh x \tanh x + \frac{2c}{\cosh x}$

(2) $\frac{dy}{dx} + 2y = e^{3x}$

$IF = e^{\int P dx}$

$\int P dx = 2x$

$IF = e^{2x}$

$y \cdot IF = \int Q \cdot IF$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}$

$ye^{2x} = \int e^{5x}$

$ye^{2x} = \frac{e^{5x}}{5} + C$

$y = \frac{e^{5x}}{5} \times \frac{1}{e^{2x}} + \frac{C}{e^{2x}}$

$y = \frac{e^{3x}}{5} + \frac{C}{e^{2x}}$

(3) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - 3x^{-1}$

$y = \int \frac{dy}{dx} = \int x + 2 - 3x^{-1}$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

(4) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$y^{-3}\frac{dy}{dx} + \frac{y^{-2}}{x} = 1 \quad \cdot *$

$z = y^{-2} \quad \frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$

multiplying * by -2

$-2y^{-3}\frac{dy}{dx} + \frac{-2y^{-2}}{x} = -2$

substituting

$\frac{dz}{dx} - \frac{2z}{x} = -2$

using bernoulli's theorem

$\int P dx = \int \frac{-2}{x} = -2\ln x$

$IF = e^{-2\ln x} = \frac{1}{x^2}$

$z \cdot IF = \int Q \cdot IF$

$z \cdot \frac{1}{x^2} = \int \frac{-2}{x^2}$

$z/x^2 = \cancel{2\ln x^2 + c} \ \frac{2}{x} + C$

$z = 2x + Cx^2$

recall $z = y^{-2}$

$y^{-2} = 2x + Cx^2$

$\frac{1}{y^2} = 2x + Cx^2$

$y^2 = \frac{1}{2x + Cx^2}$

$y = \sqrt{\frac{1}{2x + Cx^2}}$

⑤ $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = x \sin 3x + 4x^{-2}$

$y = \int \frac{dy}{dx} = \int x \sin 3x + 4x^{-2}$

$y = \int x \sin 3x + \int 4x^{-2}$

Solving $\int x \sin 3x$ using integration by Parts

$\cancel{\int v du - uv \int v du}$ $\int u dv = uv - \int v du$

$\int x \sin 3x = -\frac{\cos 3x}{3} * x - \int \sin 3x \cdot 1$

$= -\frac{x}{3} \cos 3x - \frac{1}{3} \cos 3x + c$

$y = -\frac{x}{3} \cos 3x - \frac{1}{3} \cos 3x - \frac{2}{x} + c$

⑥ $(x^3 + xy^2) \frac{dy}{dx} = 2y^3$

$y = vx \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$

$v + x \frac{dv}{dx} = \frac{2v^3 x^3}{x^3 + x^3 v^2}$

$v + x \frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v + v^3}{1+v^2}$

$= \frac{3v^3 - v}{1+v^2}$

$\frac{1+v^2}{3v^3 - v} dv = \frac{1}{x} dx$

using Partial integration

$\frac{1+v^2}{v(v-1)(v+1)} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$\frac{1+v^2}{v(v-1)(v+1)} = \frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}$

$\int \frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} dv = \int \frac{1}{x} dx$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + \ln c$

$\ln\left[\frac{(v-1)(v+1)}{v}\right] = \ln(xc)$

$\frac{v^2 - 1}{v} = Cx$

Recall $v = y/x$

$\therefore y - \frac{x^2}{y} = x^2 C$