ENG MATHS ASSIGNMENT II

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Matric No: 18/ENG05/007 — Biomedical engineering
Dept:

(a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$\cancel{\frac{dy}{dx} = 2\sinh x - y\tanh x}$

$P = \tanh x \quad Q = 2\sinh x$

$\int p\,dx = \int \tanh x\,dx = \int \frac{\sinh x}{\cosh x}\,dx$

Let $u = \cosh x \quad dx = \frac{dy}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{dy}{\sinh x}$

$\int \frac{1}{u}\,du = \ln u = \ln\cosh x$

$If = e^{\int p\,dx} = e^{\ln\cosh x}$

$If = \cosh x$

then $y\,If = \int Q \cdot If\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x\cos x = \sin(2x)$

$2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\,dx$

$y \cdot \cosh x = \frac{1}{2} \cdot 2\cosh 2x + C$

$\cosh x\,y = \cosh 2x + C$

$\boxed{y = \frac{\frac{\cosh 2x}{2} + C}{\cosh x}}$

$y = \frac{\cosh 2x + 2C}{\cosh x}$

Let $2C = A$

$\therefore y = \frac{\cosh 2x + A}{\cosh x}$

(b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int p\,dx = 2x \quad Q = e^{3x}$

$If = e^{\int p\,dx} = e^{2x}$

$y \cdot If = \int Q\,If\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$\cancel{y \cdot e^{2x} = \frac{e^{5x}}{5} + C}$

$y \cdot e^{2x} = \frac{e^{5x}}{5} + C$

$y = \frac{\frac{e^{5x}}{5}}{e^{3x}} + \frac{C}{e^{2x}}$

$y = \frac{e^{5x}}{5e^{2x}} + \frac{C}{e^{2x}}$

$y = \frac{1}{5}\frac{e^{5x}}{e^{2x}} + \frac{C}{e^{2x}}$

$y = \frac{1}{5} \cdot e^{5x-2x} + Ce^{-2x}$

$y = \frac{1}{5} \cdot e^{3x} + Ce^{-2x}$

$\boxed{y = \frac{e^{3x}}{5} + Ce^{-2x}}$

c. $x\,\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = \frac{x^2 + 2x - 3}{x}$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$dy = \left(x + 2 - \frac{3}{x}\right)dx$

$\int dy = \int \left(x + 2 - \frac{3}{x}\right)dx$

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$$\therefore \boxed{y = \frac{x^2}{2} + 2x - 3\ln x + c}$$

(d) $dy/dx + y/x = y^3$

divide through with y^3

$y^{-3} dy/dx + 1/x \, y^{1-3} = 1$ — (i)

$y^{-3} dy/dx + \frac{1}{x} y^{-2} = 1$ — (ii)

$z = y^{-2}$; $\frac{dz}{dx} = (1-3) y^{1-3} \frac{dy}{dx} = -2y^{-3}\frac{dy}{dx}$ (iii)

multiply both sides of equ (ii) by (-2)

$-2y^{-3}\frac{dy}{dx} - 2y^{-2}\frac{1}{x} = -2$ — (iv)

and $\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$

sub equ (ii) & (iii) to (iv)

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$\therefore P = -2/x$, $Q = -2$

$\int P dx = -2\ln x$. $IF = e^{\int P dx} = e^{-2\ln x} = x^{-2}$

$z\,IF = \int Q \cdot IF \, dx$

$z \cdot x^{-2} = \int -2x^{-2} dx$

$zx^{-2} = \frac{-2x^{-1}}{-1} dx$

$zx^{-2} = 2x^{-1} + c$

$z = 2x^{-1}/x^{-2} + c/x^{-2}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$\therefore 1/y^2 = x(2+cx)$

$\therefore y^2 = 1/x\,(2+cx)$

$$\boxed{y = \sqrt{1/x\,(2+cx)}}$$

(e) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = \frac{x^3 \sin 3x + 4}{x^2}$

$\frac{dy}{dx} = x \sin 3x + 4x^{-2}$

$\int \frac{dy}{dx} = \int x \sin 3x + \int 4x^{-2}$

$$\boxed{y = \frac{\sin 3x - 3x\cos(3x)}{9} + 4\ln x^{-1} + c}$$

(f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$dy/dx = 2y^3/(x^3 + xy^2)$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore dy/dx = v + x\frac{dv}{dx} = 2y^3/x^3 + xy^2$

$v + x\,dv/dx = \frac{2(vx)^3}{x^3 + x(vx)^2}$

$v + x\,dv/dx = \frac{2v^3x^3}{x^3 + xv^2x^2}$

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$$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3 + x^3v^2}$$

$$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$$

Lcm

$$x\frac{dv}{dx} = \frac{2v^3 - v(1+v^2)}{(1+v^2)}$$

$$x\frac{dv}{dx} = \frac{2v^3 - v - v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

$$\int \frac{1+v^2}{v^3 - v}\frac{dv}{dx} = \int \frac{1}{x}$$

$$\int \frac{1+v^2}{v^3 - v}\,dv = \int \frac{1}{x}\,dx$$

$$v(v-1)(v+1) = v^3 - v$$

$$\therefore \frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$$

if $v = 1$

$$1 + 1^2 = B(1)(2)$$

$$2 = B \quad \therefore B = 1$$

if $v = -1$

$$1 + (-1)^2 = C(-1)(-1-1)$$

$$2 = 2C$$

$$\therefore C = 1$$

if $v = 0$

$$1 + (0)^2 = A(0-1)(0+1)$$

$$1 = A(-1)(1) \quad \therefore A = -1$$

$$\int \left[-\frac{1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv = \int \frac{1}{x}\,dx$$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$$

$$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$$

$$\frac{v^2 - 1}{v} = Ax$$

$$y = vx \quad \therefore v = y/x$$

$$\frac{(y/x)^2 - 1}{(y/x)} = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax \cdot y/x$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(Ay + 1)$$

$$y = \sqrt{x^2(Ay + 1)}$$

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