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18/EH/Co/1004
Chemical Eng.
EHG 284.

a $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x \quad Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x\,dx}{\cosh x}$

$\cosh x = u$

$\int \frac{\sinh x\,dx}{u}$

$u = \cosh x$

$\frac{du}{dx} = \sinh x \quad dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \frac{du}{\sinh x} \quad \therefore \int \frac{1}{u} du$

$= \ln u = \ln\cosh x$

$IF = e^{\int P dx} = e^{\ln\cosh x}$

$IF = \cosh x$

Then; $y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cosh x\,dx$

and $2\sin x\cos x = \sin(2x)$

$\therefore 2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\,dx$

$y\cosh x = \frac{1}{2}\cosh 2x + c$

$\cosh x\, y = \frac{\cosh 2x}{2} + c$

$y = \frac{\frac{\cosh 2x}{2} + c}{\cosh x}$

$\therefore y = \frac{\cosh 2x + 2c}{\cosh x}$

Let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int P\,dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P dx} = e^{2x}$

$\therefore y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + c$

$y = \frac{1/5\, e^{5x} + c}{e^{2x}}$

$y = \frac{1}{5} e^{3x} + Ce^{-2x}$

c) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\therefore \int \frac{dy}{dx} dx = \int x + 2 - \frac{3}{x}\,dx$

$\therefore y = \frac{x^2}{2} + 2x - 3\ln x + C$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} y^{-3} + \frac{y^{-2}}{x} = 1 \quad -(1)$

$z = y^{1-n}$

$n = 3$

$z = y^{1-3} \therefore z = y^{-2}$

$\therefore \frac{dz}{dy} = -2y^{-3}\frac{dy}{dx} \quad -(3)$

multiply eq1 by $\frac{1-n}{2}$

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2 \quad -(4)$

and $\frac{dz}{dy} = \frac{-2y^{-3}dy}{dx}$

Sub eq 2 & 3 into 4

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$\therefore P = \frac{-2}{x} \quad \therefore Q = -2$

$\int P\,dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF\,dx$

$z \cdot x^{-2} = \int -2x^{-2}\,dx$

$= -2\int x^{-2}\,dx$

$= \frac{+2x^{-1}}{-1} + c$

$z \cdot x^{-2} = -2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + Cx^2$

$z = x(2 + Cx).$

$z = y^{-2}$

$y^{-2} = x(2 + Cx)$

$\frac{1}{y^2} = x(2 + Cx)$

$y^2 = \frac{1}{x(2 + Cx)}$

$y = \sqrt{\frac{1}{x(2 + Cx)}}$

$y = \frac{1}{\sqrt{x(2 + Cx)}}$

e) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

$\frac{dy}{dx} = x\sin 3x + \frac{4}{x^2}$

$\int \frac{dy}{dx} = \int x\sin 3x + \int 4x^{-2}$

$= \frac{-1}{3}\cos 3x + \int \frac{1}{3}\cos 3x + \frac{4x^{-1}}{-1}$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{9} - \frac{x\cos 3x}{3} - \frac{4}{x}$

f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$\therefore x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$

$\frac{1+v^2}{v^3 - v}\,dv = \frac{1}{x}\,dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$$

$v = 1 \quad \therefore \quad 1 + 1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$2 = 2c \quad \therefore c = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(v+1)$

$1 = A(-1)(1)$

$1 = A - 1 \quad \therefore A = -1$

$$\int \left[\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\right] dv = \int dx \frac{1}{x}$$

$$\int \frac{-1}{v}\, d + \int \frac{1}{v-1}\, dv - \int \frac{1}{v+1}\, dv = \int \frac{1}{x}\, dv$$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + C$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$$\frac{v^2 - 1}{v} = Ax$$

$y = vx \therefore v = \frac{y}{x}$

$$\therefore \frac{\left(\frac{y}{x}\right)^2 - 1}{y/x} = Ax$$

$$\frac{y^2}{x^2} - 1 = A\cancel{x} \cdot \frac{y}{\cancel{x}}$$

$$\frac{y^2}{x^2} - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2 y + x^2$

$\cancel{y^2 = A}$

$y^2 = x^2(Ay + 1)$