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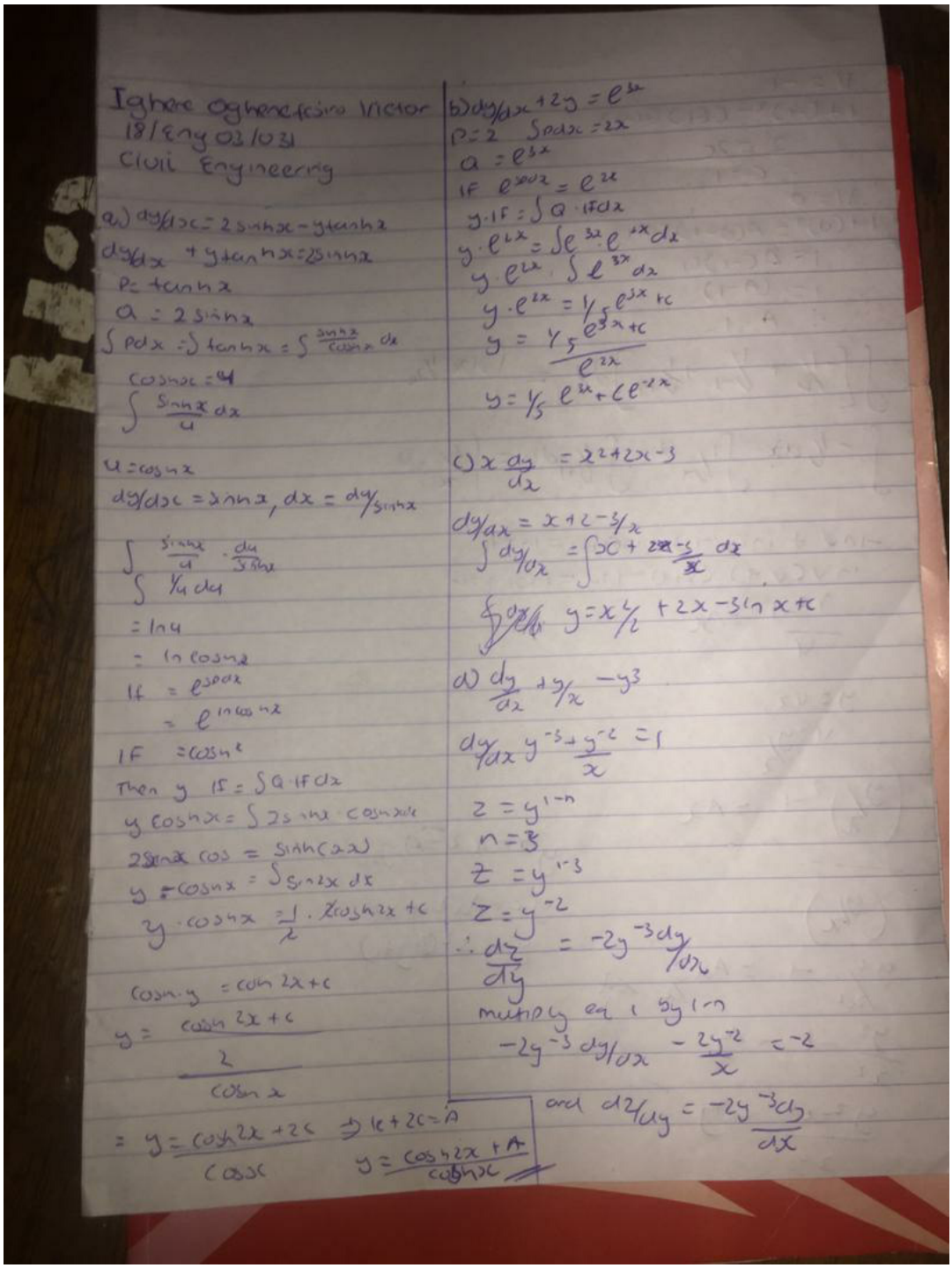

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a) $dy/dx = 2\sinh x - y\tanh x$

$dy/dx + y\tanh x = 2\sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x$

$du/dx = \sinh x, \; dx = du/\sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int 1/u \, du$

$= \ln u$

$= \ln \cosh x$

$IF = e^{\int P dx}$

$= e^{\ln \cosh x}$

$IF = \cosh x$

Then $y \cdot IF = \int Q \cdot IF \, dx$

$y \cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \, dx$

$y \cdot \cosh x = \frac{1}{2} \cdot 2\cosh 2x + c$

$\cosh x \cdot y = \cosh 2x + c$

$y = \frac{\cosh 2x + c}{\cosh x}$

$= y = \frac{\cosh 2x + 2c}{\cosh x} \Rightarrow k + 2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$IF \; e^{\int 2 dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot e^{2x} = \int e^{3x} e^{2x} dx$

$y \cdot e^{2x} \int e^{3x} dx$

$y \cdot e^{2x} = 1/5 e^{5x} + c$

$y = \frac{1/5 e^{3x} + c}{e^{2x}}$

$y = 1/5 e^{3x} + ce^{-2x}$

c) $x \frac{dy}{dx} = x^2 + 2x - 3$

$dy/dx = x + 2 - 3/x$

$\int dy/dx = \int x + 2x - \frac{3}{x} dx$

$y = x^2/2 + 2x - 3\ln x + c$

d) $\frac{dy}{dx} + y/x = y^3$

$dy/dx \, y^{-3} + \frac{y^{-2}}{x} = 1$

$z = y^{1-n}$

$n = 3$

$z = y^{1-3}$

$z = y^{-2}$

$\therefore \frac{dz}{dy} = -2y^{-3} dy/dx$

multiply eq 1 by $1-n$

$-2y^{-3} dy/dx - \frac{2y^{-2}}{x} = -2$

and $dz/dy = -2y^{-3} \frac{dy}{dx}$

$v = -1$

$1 + (-1)^2 = (A+1)(-1-1)$

$2 = 2C$

$\therefore C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = (A-1)$

$\therefore A = 1$

$$\int\left[-\tfrac{1}{v} + \tfrac{1}{v-1} + \tfrac{1}{v+1}\right]dv = \int dx\, \tfrac{1}{x}$$

$$\int -\tfrac{1}{v}\,dx \int \tfrac{1}{v-1}\,dv + \int \tfrac{1}{v+1}\,dv = \int \tfrac{1}{x}\,dv$$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$

$\ln v(v-1)(v+1) - \ln v = \ln x + \ln c$

$\frac{v^2-1}{v} = Ax$

$y = vx$

$v = y/x$

$\frac{(y/x)^2 - 1}{(y/x)} = Ax$

$\frac{y^2/x^2 - 1}{y/x} = Ax \cdot \frac{1}{x}$

$\frac{y^2}{x^2} - 1 = Ay$

$\frac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ay x^2$

$y^2 = Ax^2 y + x^2$

$y^2 = x^2(Ay + 1)$

$x\frac{dz}{dy} - 2z = -2$

$\therefore P = -2/x$

$Q = -2$

$\int P dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF\, dx$

$z \cdot x^{-2} = \int -2x^{-2} dx$

$= -2\int x^{-2} dx$

$= \frac{-2x^{-1}}{-1} + c$

$2x^{-2} = 2x^{-1} + c$

$z = \frac{2x}{x^2} + \frac{c}{x^2}$

$z = 2x + cx^2$

$z = xc_2 + cx$

$z = y^{-2}$

$y^{-2} = xc_2 + cx$

$\frac{1}{y^2} = xc_2 + cx$

$y^2 = \frac{1}{xc_2 + cx}$

$\therefore y = \sqrt{\frac{1}{\int xc_2 + cx}}$

$y = \frac{1}{\sqrt{\int xc_2 + cx}}$

e) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

$dy/dx = x\sin 3x + \frac{4}{x^2}$

$\int dy/dx = \int x\sin 3x + \int 4x^{-3}$

$= \frac{1}{3}\cos 3x - \int \frac{1}{3}\cos 3x \cdot 4x^{-1}$

$= -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-2}$

$y = \frac{\sin 3x}{9} - \frac{x\cos 3x}{3} - \frac{4}{x}$

f) $(x^3 + xy^2)\frac{dy}{dx} = -2y^3$

$y = vx$

$dy/dx = v + x\,dv/dx$

$v + x\,dv/dx = \frac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$x\,dv/dx = \frac{2v^3}{1+v^2} - v$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x\,dv/dx = x^3$

$\frac{1+v^3}{v^3 - v}\cdot dv = 1/x\, dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^3}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$v = 1$

$1 + 1^2 = B(1)(2)$

$2 = 2B$

$B = 1$