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18/ENG08/024
Biomedical Eng

a) $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

$\cosh x = u$

$\int \frac{\sinh x}{u}\,dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x \qquad dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u}\,du = \ln u = \ln \cosh x$

$IF = e^{\int P\,dx} = e^{\ln \cosh x}$

$IF = \cosh x$

then $y\,IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x\cos x = \sin 2(x)$

$\therefore 2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\,dx$

$y \cdot \cosh x = \frac{1}{x} \cdot x \cosh 2x + C$

$\cosh x\,y = \cosh 2x + C$

$y = \frac{\frac{\cosh 2x + C}{2}}{\cosh x}$

$y = \frac{\cosh 2x + 2C}{\cosh x}$

Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \qquad \int P\,dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P\,dx} = e^{2x}$

$\therefore y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$

$y = \frac{\frac{1}{5} e^{5x} + C}{e^{2x}}$

c) $2\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\therefore \int \frac{dy}{dx} = \int x + 2 - \frac{3}{x}\,dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} y^{-3} + \frac{y^{-2}}{x} = 1 \quad \cdots (1)$

$Z = y^{1-n}, \quad n = 3$

$Z = y^{1-3} = Z = y^{-2} \quad \cdots (2)$

$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx} \cdots (3)$

Then multiply eqn 1 by -2

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$

and $\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$

Sub eqn 2 & 3 into 4

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$\therefore P = -2/x, \quad Q = -2$

$\int P\,dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

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2. $If = \int Q \cdot If \cdot dx$

$z = x^{-2} = \int -2x^{-2}\,dx$

$= \dfrac{-2x^{-1}}{-1} + c$

$z \cdot x^{-2} = 2x^{-1} + C$

$z = \dfrac{2x^{-1}}{x^{-2}} + C/x^{-2}$

$z = 2x + Cx^2$

$z = x(2 \cancel{+} Cx)$

$z = y^{-2}$

$y^{-2} = x(2 + Cx)$

$y^2 = 1/x\,(2x + Cx)$

$\therefore y = \sqrt{1/x(2+Cx)}$

$y = \dfrac{1}{\sqrt{x(2+Cx)}}$

e) $x^2\, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x^2 \sin 3x + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + 1\; 4x^{-1}/-1$

$= \dfrac{-x \cos 3x}{3} + \dfrac{\sin 3x}{9} - 4x/x$

f $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x\, dv/dx$

$v + x\, dv/dx = \dfrac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\, dv/dx = x^3(2v^3)/x^3(1+v^2)$

$x\, dv/dx = 2v^3/1+v^2$

$= \dfrac{2v^3 - v(1+v^2)}{1+v^2} = \dfrac{2v^3 - v - v^3}{1+v^2}$

$x = dv/2 = v^3$

$\dfrac{1+v^2}{v^3 - v}\, dv = 1/x\, dx$

$v(v-1)(v+1) = v^3 - v$

$\dfrac{1+v^2}{v^3 - v} = \dfrac{A}{v} + \dfrac{B}{v-1} + C/v+1$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1)$
$+ C(v)(v-1) \quad v = 1$

$1 + 1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$2 = 2C$

$\therefore C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1 \quad \therefore A = -1$

$\int [-1/v + 1/v-1 + 1/v+1]\, dv = \int dx\, 1/x$

$\int -1/v\, d \int 1/v-1 + dv + \int 1/v+1\, dv =$
$\int 1/x\, dv$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$\dfrac{v^2 - 1}{v} = Ax$

$y = vx \quad ; \quad v = y/x$

$\dfrac{(y/x)^2 - 1}{(y/x)} = Ax$

$\dfrac{y^2}{x^2} - 1 = Ax(y/x)$

$\dfrac{y^2}{x^2} - 1 = Ay$

$\dfrac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2 y + x^2 \quad y^2 = x^2(Ay + 1)$