Adedokun Precious Adeola
18/ENG02/006
Computer Engineering

1) $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

$\cosh x = u$

$\int \frac{\sinh x}{u}\cdot dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x \quad dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u}\cdot\frac{du}{\sinh x}$

$= \int \frac{1}{u}\cdot du$

$= \ln u$

$= \ln \cosh x$

$\int P\,dx$

$If = e^{\int \ln \cosh x}$

$= \cosh x$

$y\cdot If = \int Q\; If\cdot dx$

$y\cdot \cosh x = \int 2\sinh x\cdot\cosh x\cdot dx$

$2\sinh x\cosh x = \sinh(2x)$

$y\cdot\cosh x = \int \sinh 2x\cdot dx$

$y\cdot\cosh x = \frac{1}{2}\times 2\cosh 2x + c$

$y\cdot\cosh x = \cosh 2x + c$

$y = \frac{\cosh 2x + c}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2$

$\int P\,dx = 2x$

$Q = e^{3x}$

$If\; e^{\int P\cdot dx}$

$= e^{2x}$

$y\cdot If = \int Q\cdot If\cdot dx$

$y\cdot e^{2x} = \int e^{3x}\cdot e^{2x}\cdot dx$

$y\cdot e^{2x} = \int e^{5x}\,dx$

$y\cdot e^{2x} = \frac{1}{5}e^{5x} + c$

$y = \frac{\frac{1}{5}e^{5x} + c}{e^{2x}}$

c) $x\,dy/dx = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x}\,dx$

$y = \frac{x^2}{2} + 3x - 3\ln x + c$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx}y^{-3} + \frac{y^{-2}}{x} = 1$

$z = y^{1-n}$

$z = y^{1-3}$

$z = y^{-2}$ -- (i)

$\frac{dz}{dy} = -2y^{-3}\,dy/dx$ -- (ii)

multiply equation by $(1-n)$

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-3}}{x} = -2$

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$

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Sub equ (i) into (ii)

$\frac{dz}{dy} = \frac{2z}{x} = -2$

$\therefore P = \frac{-2}{x} \qquad Q = -2$

$\int P\cdot dx = -2\ln x$

$If = e^{-2\ln x}$

$= -2$

$z\cdot If = \int Q\cdot If\cdot dx$

$2x^{-2} = \int \cdot 2x^{-2}\cdot dx$

$2x^{-2} = \frac{-2x^{-1}}{-1} + c$

$z = \frac{2x^{-1}}{x^{2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(x + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = \frac{1}{x(2 + cx)}$

2) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

$\frac{dy}{dx} = x\sin 3x + \int 4x^{-2}$

$= \frac{1}{3}x\cos 3x + \int \frac{1}{3}\cos 3x\,1$

$\frac{4x^{-1}}{-1}$

$-\frac{x\cos 3x}{3} - \frac{-1\sin 3x}{4} - 4x^{-1}$

$y = \frac{\sin 3x}{4} - \frac{x\cos 3x}{3} - \frac{4}{x}$

b) $(x^3 + xy^2)\,dy/dx = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dy}{dx} = \frac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\frac{dy}{dx} = \frac{(2v^3)}{(1+v^2)}$

$\frac{-2v^3 - v(1+v^2)}{1+v^2}$

$\frac{-2v^3 - v - v^3}{1+v^2}$

$x\cdot du = v^3$

$\frac{1+v^2}{v^3 - v}\,du = \frac{1}{x}\,dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$

$1 + 1^2 = B(1)(2)$

$2 = 2B$

$B = 1$

$v = -1$

$1 + (-1)^2 = 1(-1)(-1-1)$

$2 = 2C$

$C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = -1A$

$A = -1$

$$\int\left(\frac{-1}{v}+\frac{1}{v-1}+\frac{1}{v+1}\right)dv=\int\frac{1}{4}\,dx$$

$-\ln v+\ln(v-1)+\ln(v+1)=\ln x$

$\ln(v-1)(v+1)-\ln v=\ln x$

$\frac{v^2-1}{v}=\ln x$

$y=vx$

$v=\frac{y}{z}$ $\quad (y/x)^2-1=Ax$

$\frac{y^2}{x^2}-1=Ax-\frac{y}{x}$

$\frac{y^2}{x^2}-1=Ay$

$\frac{y^2-x^2}{x^2}=Ay$

$y^2-x^2=Ayx^2$

$y^2=Ax^2y+x^2$

$y^2=x^2(Ay+1)$