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a) $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

$\cosh x = 4$

$\int \frac{\sinh x}{4}\,dx$

$4 = \cosh x$

$dy/dx = \sinh x \qquad dx = dy/\sinh x$

$\int \frac{\sinh x}{4} \cdot dy/\sinh x$

$\int 1/4\,du = \ln u = \ln\cosh x$

$IF \quad e^{\int P\,dx} = e^{\ln\cosh x}$

$IF = \cosh x$

Then $y\,IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x$

$2\sin x\cos x = \sin(2x)$

$2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\,dx$

$y \cdot \cosh x = \frac{1}{2} \cdot 2\cosh 2x + c$

$\cosh x\, y = \cosh 2x + c$

$y = \frac{\cosh 2x + c}{2}$

$y = \cosh 2x + 2 / \cosh x$

Let $2c = -A$

$y = \frac{\cosh 2x - A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P\,dx = 2x$

$Q = e^{2x}$

$If = e^{\int P\,dx} = e^{2x}$

$y \cdot If = \int Q \cdot If\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{3x}\,dx$

$$y = \frac{\tfrac{1}{2}e^{3x} + c}{e^{2x}}$$

c) $dy/dx = 3x^2 + 2x - 3$

$dy/dx = x + 2 - \frac{3}{x}$

$$\int dy/dx = \int x + 2 - \frac{3}{x}\, dx$$

$$y = \frac{x^2}{2} + 2x - 3\ln x + c$$

d) $dy/dx + y/x = y^3$

$dy/dx\; y^{-3} + y^{-2}/x = 1$

$z = y^{1-n} \quad n = 3$

$z = y^{1-3}, \quad z = y^{-2}$

$dz/dx = -2y^{-3}\, dy/dx$

Then multiply equ by $1-n$

$$-2y^{-3}\, dy/dx - \frac{2y^{-2}}{x} = -2$$

$$d\,z/dy = \frac{-2y^{-3}\,dy}{dx}$$

Sub equ 2 and 3 into 4

$dz/dy - z^2/x = -2$

$P = -2/x, \quad Q = -2$

$\int p\, dx - 2\ln x$

$IF = e^{-2\ln x} = x$

$z \cdot IF = \int Q \cdot IF \cdot dx$

$z \cdot x^{-2} - \int -2x^{-2}\, dx$

$$= \frac{-2x^{-1}}{-1} + c$$

$z \cdot x^{-2} = 2x^{-1} + c$

$$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$$

$z = 2x + cx^2$

$z = x(2 + x)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = \frac{1}{x(2+cx)}$

$y = \sqrt{\frac{1}{x(2+cx)}}$

$y = \frac{1}{\sqrt{x(2+x)}}$

c) $x^2 \, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x^4 \sin 3x + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + 4x^{-1}$

$= -\frac{x \cos 3x}{3} + \frac{\sin 3x}{a} - 4x^{-1}$

$y = \frac{\sin 3x}{a} - \frac{x \cos 3x}{3} - \frac{4}{x}$

f) $(x^3 + xy^2) \, dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x \, dv/dx$

$v + x \, dv/dx = 2(vx)^3 / x^3 + v^3 x^3$

$v + x \, dy/dx = \frac{x^3 (2v^3)}{x^3 (1+v^2)}$

$x \, dv/dx = \frac{2v^3}{1+v^2}$

$x \, dv = v^3$

$\frac{1+v^2}{v^3 - v} dv - \frac{1}{x} dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$, $v = 1$

$1 + 1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = c(-1)(-1-1)$

$2 = 2c$

$\therefore c = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1$

$A = -1$

$\int \left[-\frac{1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv = \int dx = \frac{1}{x}$

$\int -\frac{1}{v} + \int \frac{1}{v-1} dv + \int \frac{1}{v+1} dv = \int \frac{1}{x} dx$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$

$\ln (v-1)(v+1) - \ln v = \ln x + \ln A$

$\frac{v^2 - 1}{v} = Ax$

$y = vx$

$\frac{(y/x)^2 - 1}{(y/x)} = Ax$

$\frac{y^2}{x^2} - 1 = Ax \cdot \frac{y}{x}$

$\frac{y^2}{x^2} - 1 = Ay$

$\frac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2 y + x^2$

$y^2 = x^2(Ay + 1)$