ESUKU DICKSON EMMANUEL
19/ENG02/076
COMPUTER ENGINEERING
ENG 284 ASSIGNMENT

a) $dy/dx = 2\sinh x - y\tanh x$

$dy/dx + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int p\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

$\cosh x = u$

$\int \frac{\sinh x}{4}\,dx$

$u = \cosh x \qquad dx = \frac{dy}{\sinh x}$

$dy/dx = \sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{dy}{\sinh x}$

$\int 1/u\,du = \ln u = \ln \cosh x$

$\text{If} = e^{\int p\,dx} = e^{\ln \cosh x}$

$IF = \cosh x$

Then $y\,IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x$

Then $y\,IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x \cos x = \sin(2x)$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh \cancel{(2x)}\, 2x\,dx$

$y \cdot \cosh x = \frac{1}{\cancel{2}} \cdot \cancel{2}\cosh 2x + c$

$\cosh xy = \cosh 2x + c$

$y = \frac{\frac{\cosh 2x + c}{2}}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

Let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P\,dx = 2x$

$Q = e^{3x}$

$If = e^{3\int dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = 1/5\,e^{5x} + c$

$y = \frac{1/5\,e^{5x} + c}{e^{3x}}$

(c) $2\,dy/dx = x^e + 2x - 5$

$dy/dx = x + 2 - 3/x$

$\therefore \int dy/dx = \int x + 2 - 3/3\,dx$

$y = x^2/2 + 2x - 3\ln x + c$

(d) $dy/dx + y/x = y^3$

$dy/dx\, y^{-3} + y^{-3}/x = 1$ — (i)

$z = y^{1-n}, \quad n = 3$

$z = y^{1-3}, \quad z = y^{-2}$ — (ii)

$\therefore dz/dy = -2y^{-3}\,dy/dx$ — (iii)

Then multiply eq(i) by $(1-n)$

$-2y^{-3}\,dy/dx - 2y^{-2}/x$ — 2

and $dz/dy = -2y^{-3}dy/dx$

Sub equ(ii) and (iii) into (iv)

$dz/dy - 2x/x = -2$

$\therefore P = -2/x, \quad Q = -2$

$\int P\,dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF\,dx$

$z \cdot x^{-2} = \int -2x^{-2}\,dx$

$= \frac{-2x^{-1}}{-1} + c$

$2x^{-2} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = 1/x\,(2+cx)$
$\therefore y = \sqrt{1/x\,(2+cx)}$
$y = 1/\sqrt{x(2+cx)}$

(e) $x = dy/dx = x^3 \sin 3x + 4$
$dy/dx = x \sin 3x + 4$
$\int dy/dx = \int \overset{u}{x} \sin \overset{dv}{3x} + \int 4x^{-2}$
$= 1/3 \cos 3x + \int 1/3 \cos 3x + 4x^{-1}$
$= \dfrac{-x\cos 3x}{3} + \dfrac{\sin 3x}{9} - 4x^{-1}$

$y = \dfrac{\sin 3x}{4} - \dfrac{x\cos 3x}{3} - \dfrac{4}{x}$

(f) $(x^3 + xy^2)\, dy/dx - 2y^3$
$y = vx$
$dy/dx = v + x\, dv/dx$
$v + x\, dy/dx = \dfrac{2(vx)^3}{x^3 + v^2x^3}$
$v + x\, dy/dx = \dfrac{x^3(2v^3)}{x^3(1+v^2)}$
$x\, dv/dx = \dfrac{2v^3}{1+v^2}$
$= \dfrac{2v^3 - v(1+v^2)}{1+v^2}$
$= \dfrac{2v^3 - v - v^3}{1+v^2}$
$x\, dv = v^3$
$\dfrac{1+v^2}{v^3 - v}\, dv = 1/x\, dx$

$v(v-1)(v+1) = v^3 - v$
$\dfrac{1+v^2}{v^3-v} = \dfrac{A}{v} + \dfrac{B}{v-1} + \dfrac{C}{v+1}$
$1 + v^2 = A(v-1)(v+1) + B(v)(v+1)$
$+ c(v)(v-1),\ v - 1$
$1 + 1^2 = B(1)(2)$
$\cancel{2} = B(\cancel{2})$
$\therefore B = 1$

$v = -1$
$1 + (-1)^2 = c(-1)(-1-1)$
$\cancel{2} = \cancel{2}c$
$\therefore c = 1$
$v = 0$
$1 + (0)^2 = 4(0-1)(0+1)$
$1 = 4(-1)(1)$
$1 = A - 1$
$\therefore A = -1$
$\int \left[-1/v + \dfrac{1}{v-1} + \dfrac{1}{v+1} \right] dv = \int dx\, 1/x$
$\int -1/v\, dv + \int 1/v-1 + dv + \int \dfrac{1}{v+1}\, dv =$
$\int 1/x\, dx$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$
$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$
$\dfrac{v^2 - 1}{v} = Ax$

$y = vx \therefore v = y/x$
$\dfrac{(y/x)^2 - 1}{(y/x)} = Ax$
$y^2/x^2 - 1 = A\cancel{x} \cdot y/\cancel{x}$
$y^2/x^2 - 1 = Ay$
$\dfrac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ayx^2$
$y^2 = Ax^2y + x^2$
$y^2 = x^2(Ay + 1)$