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COURSE : ENG 282 : MATHEMATICS

MAT. NO : 18/ENG03/055

DEPT : CIVIL ENGINEERING

DATE : 18 - 03 - 2020

MATH ASSIGNMENT.

1(a). $\frac{dy}{dx} + y\tanh x = 2\sinh x$

Solution.

$dy/dx + Py = Q$ (where P and Q are functions of x)

$y \cdot IF = \int Q \cdot IF \cdot dx$

$IF = e^{\int P dx}$, where $P = \tanh x$, $Q = 2\sinh x$

$e^{\int P dx} = e^{\int \tanh x \cdot dx}$

$= e^{\ln \cosh x} = \cosh x$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \cdot dx$

$y \cdot \cosh x = 2\int \sinh x \cdot \cosh x \cdot dx$

$y \cdot \cosh x = 2\left(\sinh^2 x/2 + c\right)$

$y = 2\left(\frac{\sinh^2 x}{2} + c\right) \div \cosh x$

$y = 2\left(\frac{\sinh^2 x}{2} + c\right) \times \frac{1}{\cosh x}$

$y = \frac{(\sinh x)^2}{\cosh x} + \frac{2c}{\cosh x}$

$y = \sinh x \tanh x + 2c(\cosh x^{-1})$

(b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad , \quad Q = e^{3x}$

$IF = e^{\int P dx} = e^{\int 2 dx}$

$IF = e^{2x}$

$\therefore y \cdot IF = \int Q \cdot IF \cdot dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \cdot dx$

$y \cdot e^{2x} = \frac{e^{5x}}{5} + c$

$y = \frac{e^{3x}}{5} + ce^{-2x}$

(c) $x\frac{dy}{dx} = x^2 + 2x - 3$

Divide through by x

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

Integrate both sides

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x}$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

(d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

Divide through by y^3

$y^{-3} \cdot \frac{dy}{dx} + \frac{y^{-2}}{x} = 1$

Let $z = y^{-2}$

$\frac{dz}{dx} = -2y^{-3} \cdot \frac{dy}{dx}$

Multiply through by (-2)

$-2y^{-3}\cdot\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$ — (ii)

Substitute $\frac{dz}{dx}$ into eq. (ii)

$\therefore$

$\frac{dz}{dx} - \frac{2z}{x} = -2$

Using $z\cdot IF = \int Q\cdot IF\cdot dx$

~~$IF = e^{\int P dx} = e^{\int 2x\, dx} = e^{x^2}$~~ $IF = e^{\int P dx} = e^{\int -2/x\, dx} = e^{-2\ln x} = x^{-2}$

~~$z\cdot e^{x^2} = \int e^{x^2}\cdot -2\cdot dx$~~ $z\cdot x^{-2} = \int -2x^{-2}$

$zx^{-2} = \frac{-2x^{-2+1}}{-2+1}$

$zx^{-2} = x^{-1}$

$z = x^{-1}\cdot x^{2}$

$z = x$

$1/y^2 = x$

$y^2 = 1/x$

$y = \sqrt{1/x}$

(e) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

$\frac{dy}{dx} = \frac{x^3\sin 3x}{x^2} + 4x^{-2}$

$\frac{dy}{dx} = x\sin 3x + 4x^{-2}$

Integrate both sides

$\int\frac{dy}{dx} = \int x\sin 3x + 4\int x^{-2}$

$\int x\sin 3x = \frac{-x\cos 3x}{3} + \int \sin 3x\cdot 1$

$\int x\sin 3x = \frac{-x\cos 3x}{3} - \frac{1}{3}\cos 3x + C$

$y = \frac{-x}{3}\cos 3x - \frac{1}{3}\cos 3x - 2/x + C$

(f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

Let $y = vx$

$(x^3 + x(v^2x^2))\frac{dy}{dx} = 2v^3x^3$

$x^3(1+v^2)\frac{dy}{dx} = 2v^3x^3$

$1+v^2\frac{dy}{dx} = \frac{2v^3x^3}{x^3}$

$(1+v^2)\frac{dy}{dx} = 2v^3$

$\frac{dy}{dx} = \frac{2v^3}{1+v^2}$

$v + x\frac{dv}{dx} = \frac{2v^3}{(1+v^2)}$

$x\frac{dv}{dx} = \frac{2v^3}{(1+v^2)} - v$

$x\frac{dv}{dx} = \frac{2v^3 - v - v^3}{(1+v^2)}$

$\frac{dv}{dx} = \frac{2v^2 - v}{(1+v^2)}$