

OKEREKE FRANCIS
19/ENG05/071
MECHATRONICS ENGINEERING
ENG 284 ASSIGNMENT

a) $\frac{dy}{dx} = 2 \sinh x - y \tanh x$
 $\frac{dy}{dx} + y \tanh x = 2 \sinh x$
 $P = \tanh x$
 $Q = 2 \sinh x$
 $\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$
 $\cosh x = u$
 $\int \frac{\sinh x}{u} dx$
 $u = \cosh x \quad dx = \frac{dy}{\sinh x}$
 $\frac{dy}{dx} = \sinh x$
 $\int \frac{\sinh x}{u} \cdot \frac{dy}{\sinh x}$
 $\int \frac{1}{u} dx = \ln u = \ln \cosh x$
 $IF = e^{\int P dx} = e^{\ln \cosh x}$
 $IF = \cosh x$
 Then $y \cdot IF = \int Q \cdot IF dx$
 $y \cdot \cosh x$
 Then $y \cdot IF = \int Q \cdot IF dx$
 $y \cdot \cosh x = \int 2 \sinh x \cdot \cosh x dx$
 $2 \sin x \cos x = \sin(2x)$
 $2 \sinh x \cosh x = \sinh(2x)$
 $y \cdot \cosh x = \int \sinh(2x) dx$
 $y \cdot \cosh x = \frac{1}{2} \cdot 2 \cosh 2x + C$
 $\cosh xy = \cosh 2x + C$
 $y = \frac{\cosh 2x + C}{\cosh x}$
 $y = \frac{\cosh 2x + 2C}{\cosh x}$
 Let $2C = A$
 $y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$
 $P = 2 \quad \int P dx = 2x$
 $Q = e^{3x}$
 $IF = e^{\int P dx} = e^{2x}$
 $y \cdot IF = \int Q \cdot IF dx$

$y \cdot e^{2x} = \int e^{5x} dx$
 $y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$
 $y = \frac{1/5 e^{5x} + C}{e^{2x}}$

(c) $2 \frac{dy}{dx} = x^2 + 2x - 5$
 $\frac{dy}{dx} = x + 2 - \frac{3}{x}$
 $\therefore \int \frac{dy}{dx} = \int x + 2 - \frac{3}{x} dx$
 $y = \frac{x^2}{2} + 2x - 3 \ln x + C$

(d) $\frac{dy}{dx} + \frac{y}{x} = y^3$
 $\frac{dy}{dx} y^{-3} + y^{-3} \frac{1}{x} = 1 \quad \text{--- (i)}$
 $z = y^{1-n}, n = 3$
 $z = y^{-2}, z = y^{-2} \quad \text{--- (ii)}$
 $\therefore \frac{dz}{dy} = -2y^{-3} \frac{dy}{dx} \quad \text{--- (iii)}$
 Then multiply eq (i) by $(1-n)$
 $-2y^{-3} \frac{dy}{dx} - 2y^{-2} \frac{1}{x} = -2$
 and $\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx}$
 Sub eq (iii) and (ii) into (iv)
 $\frac{dz}{dy} - \frac{2z}{x} = -2$
 $\therefore P = -\frac{2}{x}, Q = -2$
 $\int P dx = -2 \ln x$
 $IF = e^{-2 \ln x} = x^{-2}$
 $z \cdot IF = \int Q \cdot IF \cdot dx$
 $z \cdot x^{-2} = \int -2x^{-2} dx$
 $= \frac{-2x^{-1}}{-1} + C$
 $2x^{-2} = 2x^{-1} + C$
 $z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$
 $z = 2x + Cx^2$
 $z = x(2 + Cx)$
 $z = y^{-2}$
 $y^{-2} = x(2 + Cx)$
 $\frac{1}{y^2} = x(2 + Cx)$

$$y^2 = \frac{1}{x}(2+cx)$$

$$\therefore y = \sqrt{\frac{1}{x}(2+cx)}$$

$$y = \frac{1}{\sqrt{x}(2+cx)}$$

(e) $x = \frac{dy}{dx} = x^3 \sin 3x + 4$

$$\frac{dy}{dx} = x \sin 3x + 4$$

$$\int \frac{dy}{dx} = \int x \sin 3x + 4$$

$$= \frac{1}{3} \cos 3x + \int \frac{1}{3} \cos 3x + 4x^{-1}$$

$$= \frac{-x \cos 3x}{3} + \frac{\sin 3x}{3} - 4x^{-1}$$

$$y = \frac{\sin 3x}{3} - \frac{x \cos 3x}{3} - \frac{4}{x}$$

(f) $(x^3 + xy^2) \frac{dy}{dx} - 2y^3$

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dy}{dx} = \frac{2(xy)^3}{x^3 + v^2 x^3}$$

$$v + x \frac{dy}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$x \frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$= \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$$= \frac{2v^3 - v - v^3}{1+v^2}$$

$$x dv = v^3$$

$$\frac{1+v^2}{v^3-v} dv = \frac{1}{x} dx$$

$$v(v-1)(v+1) = v^3 - v$$

$$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$$1+v^2 = A(v-1)(v+1) + B(v)(v+1)$$

$$+ C(v)(v-1), v-1$$

$$1+1^2 = B(1)(2)$$

$$2 = B(2)$$

$$\therefore B = 1$$

$$v = -1$$

$$1 + (-1)^2 = C(-1)(-1-1)$$

$$2 = 2C$$

$$\therefore C = 1$$

$$v = 0$$

$$1 + (0)^2 = 4(0-1)(0+1)$$

$$1 = 4(-1)(1)$$

$$1 = 4A - 4$$

$$\therefore A = -1$$

$$\int \left[\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv = \int \frac{dx}{x}$$

$$\int \frac{-1}{v} dv + \int \frac{1}{v-1} dv + \int \frac{1}{v+1} dv = \int \frac{1}{x} dx$$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + C$$

$$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$$

$$\frac{v^2 - 1}{v} = Ax$$

$$y = vx \therefore v = \frac{y}{x}$$

$$\frac{(\frac{y}{x})^2 - 1}{(\frac{y}{x})} = Ax$$

$$\frac{y^2/x^2 - 1}{y/x} = Ax$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2 y + x^2$$

$$y^2 = x^2 (Ay + 1)$$