a) $dy/dx = 2\sinh x - y\tanh x$

$dy/dx + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{4} dx$

$u = \cosh x \qquad dx = \frac{dy}{\sinh x}$

$du/dx = \sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{dy}{\sinh x}$

$\int 1/u \, du = \ln u = \ln\cosh x$

$If = e^{\int P dx} = e^{\ln\cosh x}$

$IF = \cosh x$

Then $y \; IF = \int Q \cdot IF \, dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$2\sin x\cos x = \sin(2x)$

$2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \, dx.$

$y \cdot \cosh x = 1/2 \cdot \cosh 2x + C$

$\cosh x \, y = \cosh 2x + C$

$y = \frac{\frac{\cosh 2x}{2} + C}{\cosh x}$

$y = \frac{\cosh 2x + 2C}{\cosh x}$

Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$If = e^{\int P dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = 1/5 e^{5x} + C$

$y = \frac{1/5 e^{5x} + C}{e^{2x}}$

c) $2 dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - 3/x$

$\therefore \int dy/dx = \int x + 2 - 3/3 \, dx$

$y = x^2/2 + 2x - 3\ln x + C$

d) $dy/dx + y/x = y^3$

$dy/dx \, y^{-3} + y^{-2}/x = 1$ ——— (I)

$Z = y^{1-n}, \; n = 3$

$Z = y^{1-3}, \; Z = y^{-2}$ ——— (II)

$\therefore dz/dy = -2y^{-3} dy/dx$ ——— (III)

Then multiply eq(I) by ~~eq(III)~~ $(1-n)$

$-2y^{-3} dy/dx - 2y^{-2}/x = -2$

and $dz/dy = -2y^{-3} dy/dx$

Sub eqn (II) & (III) into (IV)

$dz/dy - 2z/x = -2$

$\therefore P = -2/x, \; Q = -2$

$\int P dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$Z \cdot IF = \int Q \cdot IF \cdot dx$

$Z \cdot x^{-2} = \int -2x^{-2} dx$

$= \frac{-2x^{-1}}{-1} + C$

$2x^{-2} = 2x^{-1} + C$

$Z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$

$Z = 2x + Cx^2$

$Z = x(2 + Cx)$

$Z = y^{-2}$

$y^{-2} = x(2 + Cx)$

$1/y^2 = x(2 + Cx)$

$y^2 = 1/x(2 + Cx)$

$\therefore y = \sqrt{1/x(2 + Cx)}$

$y = 1/\sqrt{x(2+cx)}$

e, $x^2 \, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x^4 \sin 3x \, dv + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + 4x^{-1}$

$= \frac{-x \cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{9} - \frac{x \cos 3x}{3} - \frac{4}{x}$

f, $(x^3 + xy^2) \, dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x \, dv/dx$

$v + x \, dv/dx = \frac{2(vx)^3}{x^3 + v^2 x^3}$

$v + x \, dv/dx = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$x \, dv/dx = \frac{2v^3}{1+v^2}$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x \, dv = v^3$

$\frac{1+v^2}{v^3 - v} \, dv = 1/x \, dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1)$
$+ C(v)(v-1)$, $v = 1$

$1 + 1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$ ...

$2 = 2c$

$\therefore c = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1$

$\therefore A = -1$

$\int \left[-1/v + \frac{1}{v-1} + \frac{1}{v+1} \right] dv = \int dx \, 1/x$

$\int -1/v \, dv + \int 1/(v-1) \, dv + \int \frac{1}{v+1} \, dv =$

$\int 1/x \, du$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$\frac{v^2 - 1}{v} = Ax$

$y = vx \therefore v = y/x$

$\frac{(y/x)^2 - 1}{(y/x)} = Ax$

$y^2/x^2 - 1 = Ax \cdot y/x$

$y^2/x^2 - 1 = Ay$

$\frac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2 y + x^2$

$y^2 = x^2(Ay + 1)$.