Ologunagba Bright Toluwalope
18/ENG04/062

$\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P \cdot dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} \cdot dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} \cdot dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x$

$dx = du / \sinh x$

$\int \frac{\sinh x}{u} \cdot du / \sinh x$

$= \int \frac{1}{u} \cdot du$

$= \ln u$

$= \ln \cosh x$

$If = e^{\int P \cdot dx}$

$= e^{\int \ln \cosh x}$

$= \cosh x$

$y \cdot If = \int Q \cdot If \cdot dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \cdot dx$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \cdot dx$

$y \cdot \cosh x = \frac{1}{2} \times 2\cos 2x + C$

$y \cdot \cosh x = \cosh 2x + C$

$y = \frac{\cosh 2x + C}{\cosh x}$

(b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2$

$\int P \cdot dx = 2x$

$Q = e^{3x}$

$If = e^{\int P \cdot dx}$

$= e^{2x}$

$y \cdot If = \int Q \cdot If \cdot dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \cdot dx$

$y \cdot e^{2x} = \int e^{5x} \cdot dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$

$y = \frac{\frac{1}{5} e^{5x} + C}{e^{2x}}$

(c) $x \frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x} \, dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

(d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} y^{-3} + \frac{y^{-2}}{x} = 1$ --- (i)

$z = y^{1-n}$

$z = y^{1-3}$

$z = y^{-2}$ --- (ii)

$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx}$ --- (iii)

multiply equ by $(1-n)$

$-2y^{-3} \frac{dy}{dx} - \frac{2y^{-3}}{x} = -2$

$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx}$

sub equ (2) into

$\frac{dz}{dy} - \frac{2z}{x} =$

$\therefore P = -\frac{2}{x}$

$\int P \cdot dx = -$

$If = e^{-2\ln x}$

$= -2$

$z \cdot If = \int Q$

$z \cdot x^{-2} = \int$

$z x^{-2} = -$

$z =$

$z = 2x$

$z = x($

$z = y^{-2}$

$y^{-2} =$

$\frac{1}{y^2}$

y^2

(e) $x^2 \frac{dy}{d}$

$\frac{dy}{dx}$

$\int \frac{dy}{dx}$

$= -x$

$y = \frac{\sin}{4}$

sub equ (2) into (4)

$dz/dy - 2z/x = -2$

$\therefore P = -2/x \quad Q = -2$

$\int P \cdot dx = -2 \ln x$

$IF = e^{-2\ln x}$

$= -2$

$z \cdot IF = \int Q \cdot IF \cdot dx$

$z x^{-2} = \int -2x^{-2} \cdot dx$

$zx^{-2} = \frac{-2x^{-1}}{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = 1/x(2 + cx)$

dx

$x + c$

--- (i)

(ii)

$(1-n)$

-2

ⓔ $x^2 dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4/x^2$

$\int dy/dx = \int x \sin 3x + \int 4x^{-2}$

$= \frac{1}{3}x \cos 3x + \int \frac{1}{3} \cos 3x f$

$4x^{-1}/-1$

$= \frac{-x \cos 3x}{3} + \frac{\sin 3x}{4} - 4x^{-1}$

$y = \frac{\sin 3x}{4} - \frac{x \cos 3x}{3} - 4/x$

ⓕ $(x^3 + xy^2) dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x\, dy/dx$

$v + x\, dy/dx = \frac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\, dy/dx = \cancel{x^3}/\cancel{x^3} \left(\frac{2v^3}{(1+v^2)}\right)$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x \cdot du = v^3$

$\frac{1+v^2 \cdot du}{v^3 - u} = 1/x \cdot dx$

$v(v-1)(v+1) = v^3 - u$

$\frac{1+v^2}{v^3 - v} = A/u + B/v-1 + C/v+1$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$

$1 + 1^2 = B(1)(2)$

$2 = 2B$

$B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$2 = 2C$

$C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = -1A$

$A = -1$

$$\int\left(-\frac{1}{v}+\frac{1}{v-1}+\frac{1}{v+1}\right)du = \int \frac{1}{x}\cdot dx$$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x$$

$$\ln(v-1)(v+1) - \ln v = \ln x$$

$$\frac{v^2-1}{v} = \ln x$$

$$y = vx$$

$$v = y/x$$

$$(y/x)^2 - 1 = Ax$$

$$y^2/x^2 - 1 = Ax - y/x$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2-x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2 y + x^2$$

$$y^2 = x^2(Ay + 1)$$