IKESIANDI UJU DIVINE

18/ENG02/047

COMPUTER ENGINEERING

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Soln

1) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = \frac{x^2 + 2x - 3}{x}$

$\frac{dy}{dx} = x + 2 - 3x^{-1}$

$dy = (x + 2 - 3x^{-1})\,dx$

$\int dy = \int (x + 2 - 3x^{-1})\,dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

2) $\frac{dy}{dx} + 2y = e^{3x}$

$\frac{dy}{dx} + Py = Q$

$P = 2$

$\int P\,dx = 2x$

$I.F = e^{\int P\,dx} = e^{2x}$

$y.\,IF = \int Q.\,IF\,dx$

$y.\,e^{2x} = \int e^{3x}.\,e^{2x}\,dx$

$y.\,e^{2x} = \int e^{5x}\,dx$

$y.\,e^{2x} = \frac{e^{5x}}{5} + c$

$y = \frac{e^{5x}}{5e^{2x}} + \frac{c}{e^{2x}}$

$y = \frac{e^{3x}}{5} + \frac{c}{e^{2x}}$

$$\frac{dy}{dx} + \frac{y}{x} = y^3 \implies \frac{dy}{dx} + Py = Qy^n$$

Dividing through by y^3

$$y^{-3}\frac{dy}{dx} + \frac{y^{1-3}}{x} = 1$$

$$z = y^{1-n} = y^{1-3} = y^{-2}$$

$$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$$

multiplying through by -2

$$-2y^{-3}\frac{dy}{dx} - \frac{2y^{1-3}}{x} = -2 \implies -2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$$

Since $\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$ and $z = y^{-2}$

$$\frac{dz}{dx} - \frac{2z}{x} = -2$$

$$I.F = \int Q \cdot IF\, dx$$

$$P = -\frac{2}{x}$$

$$\int P dx = -2\ln x$$

$$I.F = e^{\int P dx} = e^{-2\ln x} = \frac{1}{x^2}$$

$$z \cdot \frac{1}{x^2} = \int -2 \cdot \frac{1}{x^2}\, dx$$

$$z \cdot \frac{1}{x^2} = \int \frac{-2}{x^2}\, dx$$

$$z \cdot \frac{1}{x^2} = \int -2x^{-2}\, dx$$

$$z \cdot \frac{1}{x^2} = \frac{-2x^{-2+1}}{-2+1} + c$$

$$z \cdot \frac{1}{x^2} = 2x^{-1} + c$$

$$\frac{1}{x^2} = \frac{1}{2x} + c$$

$$z = \left(\frac{1}{2x} + c\right) x^2$$

$$z = \frac{x}{2} + cx^2$$

$$y^{-2} = \frac{x}{2} + cx^2$$

$$\frac{1}{y^2} = \frac{x}{2} + cx^2$$

$$y^2 = \frac{2}{x} + \frac{1}{cx^2}$$

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4) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$$\frac{dy}{dx} = \frac{x^3 \sin 3x + 4}{x^2}$$

$$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$$

$$dy = (x \sin 3x + 4x^{-2})\, dx$$

$$\int dy = \int (x \sin 3x + 4x^{-2})\, dx$$

$$y = -\frac{x \cos 3x}{3} + \frac{4x^{-2+1}}{-2+1} + c$$

$$y = -\frac{x \cos 3x}{3} - 4x^{-1} + c$$

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5) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$$x(x^2 + y^2)\frac{dy}{2y^3} = dx$$

$$x^2 + \frac{y^2 dy}{2y^3} = \frac{dx}{x}$$

$$\frac{y^2 dy}{2y^3} = \left(\frac{1}{x} - x^2\right) dx$$

$$\frac{1}{2y} dy = \left(\frac{1}{x} - x^2\right) dx$$

$$\int 2y^{-1} dy = \int \left(\frac{1}{x} - x^2\right) dx$$

$$2\ln y = \ln x - \frac{x^3}{3} + c$$

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6) $\frac{dy}{dx} + y\tanh x = 2\sinh x \implies \frac{dy}{dx} + Py = Q$

$P = y\tanh x$

$$\int P dx = \int \tanh x \, dx = \frac{1}{\cosh x} = \ln \cosh x$$

$$I.F = e^{\int P dx} = e^{\ln \cosh x} = \cosh x$$

$$y \cdot IF = \int Q \cdot IF \, dx$$

$$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \, dx$$

$$y \cdot \cosh x = \int \sinh 2x \, dx$$

$$y \cdot \cosh x = \frac{\cosh 2x}{2} + c$$

$$y = \frac{\cosh 2x}{2\cosh x} + \frac{c}{\cosh x}$$

$$y = \frac{\cosh^2 x + \sinh^2 x}{2\cosh x} + \frac{c}{\cosh x}$$
