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18/ENG04/070
ELECT-ELECT.

a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x = \frac{\sinh x}{\cosh x}$

$Q = 2\sinh x$

$IF = e^{\int \frac{\sinh x}{\cosh x}}$ $u = \cosh x \Rightarrow \frac{du}{dx} = \sinh x$

$y \cdot IF = \int Q \cdot IF\, dx \,|\, IF = e^{\int \frac{f'(x)}{f(x)} dx}$

$\therefore IF = e^{\ln\cosh x} = \cosh x$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\, dx$

$y\cosh x = 2\int \sinh x \cosh x$

$y\cosh x = 2\int f(x) \cdot f'(x)$

$y\cosh x = 2\left(\frac{f(x)^2}{2} + c\right)$

$y\cosh x = \frac{2\sinh^2 x}{2} + 2c$

$y\cosh x = \sinh^2 x + 2c$

$y = \frac{(\sinh x)^2}{\cosh x} + 2c(\cosh x)^{-1}$

$y = \sinh x \tanh x + \frac{2c}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2,\ Q = e^{3x}$

$y \cdot IF = \int Q \cdot IF\, dx$

$IF = e^{\int P dx} = e^{\int 2 dx} = e^{2x}$

$ye^{2x} = \int e^{3x} \cdot e^{2x}\, dx$

$ye^{2x} = \int e^{5x} + c$

$y = e^{5x - 2x} + ce^{-2x}$

$y = e^{3x} + ce^{-2x}$

c) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$y = \int (x + 2 - 3x^{-1})\, dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$y^{-3}\frac{dy}{dx} + \frac{1}{x} \cdot y^{-2} = 1$ —— (1)

$z = y^{1-n} = y^{1-3} = y^{-2}$

$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$

(1) $\times -2$: $-2y^{-3}\frac{dy}{dx} - \frac{2}{x}y^{-2} = -2$

Subs y^{-2} for z & $-2y^{-3}\frac{dy}{dx}$ for $\frac{dz}{dx}$

$\cancel{-2y}\ \frac{dz}{dx} - \frac{2}{x}z = -2$

$\therefore P = \frac{-2}{x},\ Q = -2$

$IF = e^{\int P dx} = e^{\int -\frac{2}{x} dx}$

$= e^{-2\int \frac{1}{x} dx} = e^{-2\ln x}$

$= e^{\ln x^{-2}} = x^{-2}$

$z \cdot IF = \int Q \cdot IF\, dx$

$z \cdot x^{-2} = \int -2 \cdot x^{-2}$

$zx^{-2} = \frac{-2x^{-2+1}}{-2+1} = 2x^{-1}$

$z = 2x^{-1+2} = 2x$ | $\frac{1}{y^2} = 2x$ $\therefore y = \sqrt{1/2x}$

c) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = x\sin 3x + 4x^{-2}$

$y = \int (x\sin 3x + 4x^{-2})$

$\cancel{y = \ldots - \frac{x}{3}\cos 3x - 4x^{-1} + c}$

$\cancel{\therefore y = -\frac{x}{3}\cos 3x - 4x^{-1} + c}$

$y = \int x\sin 3x - 4x^{-1} + c$

$\Rightarrow \int x\sin 3x = -\frac{x}{3}\cos 3x + \int \sin 3x \cdot 1$

$= -\frac{x}{3}\cos 3x - \frac{1}{3}\cos 3x$

$\therefore y = -\frac{x}{3}\cos 3x - \frac{1}{3}\cos 3x - \frac{4}{x} + c$

b) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

let $y = vx$

$(x^3 + x(vx)^2)\frac{dy}{dx} = 2(vx)^3$

$x^3(1+v^2)\frac{dy}{dx} = 2v^3x^3$

$(1+v^2)\frac{dy}{dx} = 2v^3$

$\frac{dy}{dx} = \frac{2v^3}{(1+v^2)}$

$v + x\frac{dv}{dx} = \frac{2v^2}{(1+v^2)}$

$x\frac{dv}{dx} = \frac{2v^3}{(1+v^2)} - v$

$x\frac{dv}{dx} = \frac{2v^3 - v(1+v^2)}{(1+v^2)}$

$x\frac{dv}{dx} = \frac{2v^3 - v - v^3}{(1+v^2)}$

$\frac{dv}{dx} = \frac{v^3 - v}{x(1+v^2)}$

$\frac{(1+v^2)}{v^3 - v}dv = \frac{1}{x}dx$

$\int \frac{1+v^2}{v^3 - v}dv = \int \frac{1}{x}dx$

$\int \frac{1+v^2}{v^3 - v} = \ln x + c$