

$$\int \left( \frac{-1/2}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right) dv = \int \frac{1/4}{x} dx$$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x$$

$$\ln(v-1)(v+1) - \ln v = \ln x$$

$$\frac{v^2-1}{v} = \ln x$$

$$y = vx$$

$$v = y/x$$

$$(y/x)^2 - 1 = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax - y/x$$

$$\frac{y^2}{x^2} - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2 y + x^2$$

$$y^2 = x^2 (Ay + 1)$$

sub eqn (v) into (iv)

$$\frac{dz}{dy} - \frac{2z}{y} = -2$$

$$P = -\frac{2}{y} \quad Q = -2$$

$$\int P \cdot dx = -2 \ln x$$

$$I.F. = e^{-2 \ln x}$$

$$= x^{-2}$$

$$z \cdot I.F. = \int Q \cdot I.F. \cdot dx$$

$$z \cdot x^{-2} = \int -2x^{-2} \cdot dx$$

$$z \cdot x^{-2} = \frac{-2x^{-1}}{-1} + C$$

$$z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$$

$$z = 2x + Cx^2$$

$$z = x(2 + Cx)$$

$$z = y^{-2}$$

$$y^{-2} = x(2 + Cx)$$

$$\frac{1}{y^2} = x(2 + Cx)$$

$$y^2 = \frac{1}{x(2 + Cx)}$$

$$\textcircled{1} \quad x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$$

$$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$$

$$\int \frac{dy}{dx} = \int x \sin 3x + \int 4x^{-2}$$

$$= \frac{1}{3} x \cos 3x + \int \frac{1}{3} \cos 3x + \frac{4x^{-1}}{-1}$$

$$= \frac{-x \cos 3x}{3} - \frac{\sin 3x}{4} - 4x^{-1}$$

$$y = \frac{\sin 3x}{4} - \frac{x \cos 3x}{3} - \frac{4}{x}$$

$$\textcircled{2} \quad (x^3 + xy^2) \frac{dy}{dx} = 2xy^3$$

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{2(vx)^3}{x^3 + v^2 x^3}$$

$$v + x \frac{dv}{dx} = \frac{2v^3}{x^3(1+v^2)}$$

$$= \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$$= \frac{2v^3 - v - v^3}{1+v^2}$$

$$= \frac{v^3 - v}{1+v^2}$$

$$x \, du = v^3$$

$$1 + v^2 - du = \frac{1}{x} dx$$

$$\frac{v(v-1)(v+1)}{v^3-v} = \frac{1}{x} dx$$

$$1 + v^2 = A \frac{1}{v} + B \frac{1}{v-1} + C \frac{1}{v+1}$$

$$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$$

$$1 + v^2 = A(v^2-1) + B(v^2+v) + C(v^2-v)$$

$$1 + v^2 = B(1)(2)$$

$$2 = 2B$$

$$B = 1$$

$$v = -1$$

$$1 + (-1)^2 = C(-1)(-1-1)$$

$$2 = 2C$$

$$C = 1$$

$$v = 0$$

$$1 + (0)^2 = A(0-1)(0+1)$$

$$1 = A(-1)(1)$$

$$1 = -A$$

$$A = -1$$

$$\frac{dy}{dx} = 2 \sinh x - y \tanh x$$

$$\frac{dy}{dx} + y \tanh x = 2 \sinh x$$

$$P = \tanh x$$

$$Q = 2 \sinh x$$

$$\int P \cdot dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} \cdot dx$$

$$\cosh x = u$$

$$\int \frac{\sinh x}{u} \cdot dx$$

$$u = \cosh x$$

$$\frac{du}{dx} = \sinh x$$

$$dx = \frac{du}{\sinh x}$$

$$\int \frac{\sinh x \cdot \frac{du}{\sinh x}}{u}$$

$$= \int \frac{1}{u} \cdot du$$

$$= \ln u$$

$$= \ln \cosh x$$

$$I_f = e^{\int P \cdot dx}$$

$$= e^{\int \ln \cosh x}$$

$$= \cosh x$$

$$y \cdot I_f = \int Q \cdot I_f \cdot dx$$

$$y \cdot \cosh x = \int 2 \sinh x \cdot \cosh x \cdot dx$$

$$2 \sinh x \cdot \cosh x = \sinh(2x)$$

$$y \cdot \cosh x = \int \sinh 2x \cdot dx$$

$$y \cdot \cosh x = \frac{1}{2} \times 2 \cos 2x + C$$

$$y \cdot \cosh x = \cos 2x + C$$

$$y' = \frac{\cos 2x + C}{\cosh x}$$

$$(b) \frac{dy}{dx} + 2y = e^{2x}$$

$$P = 2$$

$$\int P \cdot dx = 2x$$

$$Q = e^{2x}$$

$$I_f = e^{\int P \cdot dx}$$

$$= e^{2x}$$

$$y \cdot I_f = \int Q \cdot I_f \cdot dx$$

$$y \cdot e^{2x} = \int e^{2x} \cdot e^{2x} \cdot dx$$

$$y \cdot e^{2x} = \int e^{4x} \cdot dx$$

$$y \cdot e^{2x} = \frac{1}{4} e^{4x} + C$$

$$y = \frac{\frac{1}{4} e^{4x} + C}{e^{2x}}$$

$$(c) x \frac{dy}{dx} = x^2 + 2x - 3$$

$$\frac{dy}{dx} = x + 2 - \frac{3}{x}$$

$$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x} \cdot dx$$

$$y = \frac{x^2}{2} + 2x - 3 \ln|x| + C$$

$$(d) \frac{dy}{dx} + \frac{y}{x} = y^3$$

$$\frac{dy}{dx} y^{-3} + \frac{y^{-2}}{x} = 1 \quad \dots (i)$$

$$z = y^{1-n}$$

$$z = y^{1-3}$$

$$z = y^{-2} \quad \dots (ii)$$

$$\frac{dz}{dy} = -2y^{-3} \cdot \frac{dy}{dx} \quad \dots (iii)$$

multiply eqn by (i-ii)

$$-2y^{-3} \frac{dy}{dx} - \frac{2y^{-3}}{x} = -2$$

$$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx}$$