EGBUNU HIKMAT IGANYA
18/ENG03/025
CIVIL ENGINEERING

a). $dy/dx + y\tanh x = 2\sinh x$

$P = \tanh x$, $Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{4} dx$

$u = \cosh x \quad dx = \frac{dy}{\sinh x}$

$du/dx = \sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{dy}{\sinh x}$

$\int 1/u \, du = \ln u = \ln\cosh x$

$IF = e^{\int pdx} = e^{\ln\cosh x}$

$IF = \cosh x$

Then $y \cdot IF = \int Q \cdot IF dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \cdot dx$

$2\sin x\cos x = \sin(2x)$

$2\sinh x\cosh x = -\sinh(2x)$

$y \cdot \cosh x = \int \sinh x \, 2x \, dx$

$y \cdot \cosh x = 1/2 \cdot 2\cosh 2x + C$

$\cosh xy = \cosh 2x + C$

$y = \frac{\frac{\cosh 2x}{2} + C}{\cosh x}$

$y = \frac{\cosh 2x + 2C}{\cosh x}$

Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

(b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = 1/5 e^{5x} + c$

$y = 1/5 e^{5x-2x} + ce^{-2x}$

$y = 1/5 e^{3x} + K$; where $K = ce^{-2x}$.

(c) $x \, dy/dx = x^2 + 2x - 3$

$dy/dx = \frac{x^2 + 2x - 3}{x}$

$dy = \left(\frac{x^2 + 2x - 3}{x}\right) dx$

$dy = (x + 2 - 3/x) dx$

$\int dy = \int (x + 2 - 3/x) dx + K$

$\therefore y = \frac{x^2}{2} + 2x - 3\log_e x + K$

(d) $dy/dx + y/x = y^3$

$dy/dx \, y^{-3} + y^{-3}/x = 1$ —— (i)

$z = y^{1-n}$, $n = 3$

$z = y^{1-3}$, $z = y^{-2}$ —— (ii)

$\therefore dz/dy = -2y^{-3} dy/dx$ —— (iii)

Multiply eqn(i) by $(1-n)$

$-2y^{-3} dy/dx - 2y^{-3}/x = -2$

and $\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ —— (iv)

Subs eq (ii) & (iii) into (iv)

$dz/dy = 2x/x = -2$

$\therefore P = -2/x, \quad Q = -2$

$\int P dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$Z \cdot IF = \int Q \cdot IF \cdot dx$

$z \cdot x^{-2} = \int -2x^{-2} dx$

$= \frac{-2x^{-1}}{-1} + c$

$2x^{-3} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$Z = x(2+cx)$

$Z = y^{-2}$

$y^{-2} = x(2+cx)$

$1/y^2 = x(2+cx)$

$y^2 = \frac{1}{x(2+cx)}$

$\therefore y = \sqrt{\frac{1}{x(2+cx)}}$

$y = \frac{1}{\sqrt{x(2+cx)}}$

(e) $x^2 \, dy/dx = x^2 \sin 3x + 4$

$dy/dx = x\sin 3x + 4$

$\int dy/dx = \int x^4 \sin 3x + \int 4x^{-4}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + 4x^{-1}$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{4} - 4x^{-1}$

$= \frac{\sin 3x}{4} - \frac{x\cos 3x}{3} - \frac{4}{x}$

(f) $(x^3 + xy^2)\, dy/dx - 2y^3$

$y = vx$

$dy/dx = v + x\, dv/dx$

$v + x\, dv/dx = \frac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\, dy/dx = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$x\, dv/dx = \frac{2v^3}{(1+v^2)}$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$\cancel{x}\, dv = v^3$

$\frac{1+v^2}{v^3 - v}\, dv = 1/x\, dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1), \ v-1$

$1+1^2 = B(1)(2)$

$2 = 2B \quad \therefore B = 2/2 = 1$

Let $v = -1$

$1+(-1)^2 = C(-1)(-1-1)$

$2 = 2C \quad \therefore C = 2/2 = 1$

Let $v = 0$

$1+(0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1 \quad \therefore A = -1.$

$\int\left[\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\right] dv = \int dx\, 1/x$

$\int -1/v\, dv + \int \frac{1}{v-1} + dv + \int \frac{1}{v+1} dv = \int 1/x\, du$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + C$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$\frac{v^2-1}{v} = 4x$

$y = vx \therefore v = y/x$

$$\frac{(y/x)^2 - 1}{(y/x)} = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax \cdot \frac{y}{x}$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(Ay + 1)$$