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(a) $\frac{dy}{dx} = 2\sinh x = y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x \quad dx = \frac{dy}{\sinh x}$

$\frac{du}{dx} = \sinh x \cdot \frac{dy}{\sinh x}$

$\int \frac{1}{u} du = \ln u = \ln\cosh x$

$IF = e^{\int P dx} = e^{\ln\cosh x}$

$IF = \cosh x$

Then $y \cdot IF = \int Q \cdot IF dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x dx$

$2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \frac{1}{2} \cdot 2\cosh 2x + c$

$y \cosh x\, y = \cosh 2x + c$

$y = \dfrac{\frac{\cosh 2x + c}{2}}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

(b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = \cancel{1/5 e^{5x} + c}\ \frac{e^{5x}}{5} + c$

$\cancel{y = \dfrac{\frac{1}{5e^{5x}} + c}{e^{2x}}}$

$y = \left(\frac{e^{5x}}{5} + c\right) \div e^{2x}$

$y = \frac{e^{3x}}{5} + e^{-2x}c$

(c) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - 3x^{-1}$

$\int \frac{dy}{dx} = \int x + 2 - 3x^{-1} dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

d $\frac{dy}{dx} + \frac{y}{x} = y^3$

Comparing to $\frac{dy}{dx} + Py = Qy^n$

$Q = 1 \quad P = x^{-1} \quad = n = 3$

divide through by $y^3$

$y^{-3}\frac{dy}{dx} + \frac{y^{-2}}{x} = 1$ equ (1)

let $z = y^{-2}$ put $z = y^{-2}$ in equ (1)

$= y^{-3}\frac{dy}{dx} + \frac{z}{x} = 1$

differentiate $z$ with respect to $x$

$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$ — (2)

multiply through by equ (1) by $-2$

$-2y^{-3}\frac{dy}{dx} + \frac{-2y^{-2}}{x} = -2$

Substitute $z = y^{-2}$ and $\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$

in equation (2)

$\frac{dz}{dx} - \frac{2z}{x} = -2$ (3)

Applying integrating factor to equ (3)

$\frac{dy}{dx} + Py = Q$

$P = \frac{-2}{x}$ $Q = -2$

$\int p\,dx = -2\ln x$

$e^{\int p\,dx} = e^{-2\ln x}$ — IF

$IF = \frac{1}{x^2}$

$z \cdot IF = \int Q \cdot IF\,dx$

$z \cdot \frac{1}{x^2} = \int -2 \cdot \frac{1}{x^2}\,dx$

$z \cdot \frac{1}{x^2} = \int \frac{-2}{x^2}\,dx$

$z \cdot \frac{1}{x^2} = \int \frac{-2}{x^2}\,dx$

$z \cdot \frac{1}{x^2} = \int -2x^{-2}\,dx$

$\frac{z}{x^2} = \frac{-2x^{-2+1}}{-2+1} + C$

$\frac{z}{x^2} = 2x^{-1} + C$

multiplying through by $x^2$

$z = 2x + x^2C$

recall that $z = y^{-2}$

$y^{-2} = 2x + x^2C$

$y = \frac{1}{\sqrt{2x + x^2C}}$

(e) (e $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

$\frac{dy}{dx} = x\sin 3x + 4x^{-2}$

$\int\frac{dy}{dx} = \int x \overset{u}{} \sin 3x \overset{dv}{} + \int 4x^{-2}$

y. $u = x$ $dv = \sin 3x$

$du = 1$ $v = \frac{-\cos 3x}{3}$

$uv - \int v\,du =$

$= x \cdot -\frac{\cos 3x}{3} - \int 1 \cdot \frac{-\cos 3x}{3} \cdot +$

$= -x\frac{\cos 3x}{3} - \cancel{\int \sin 3x} \int \frac{-\cos 3x}{3}$

$= \frac{-x\cos 3x}{3} + \frac{\cos 3x}{3}$

$$-\frac{x\cos 3x}{3} - \frac{\sin 3x}{9}$$

$$= -\frac{x\cos 3x}{3} - \frac{\sin 3x}{9} - 4x^{-1} + C$$

(ii) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\,dv/dx$

$$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$= \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$x\,dv = v^3$

$$\frac{1+v^2}{v^3-v}\,dv = \frac{1}{x}\,dx$$

$v(v-1)(v+1) = v^3 - v$

$$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$1+v^2 = A(v-1)(v+1) + B(v)(v+1)$
$+ C(v)(v-1)$, $v=1$

$1+1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$\therefore C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1$

$A = -1$

$$\int\left[\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\right]dv = \int dx\,\frac{1}{x}$$

$$\int -\frac{1}{v}\,dv + \int\frac{1}{v-1}\,dv + \int\frac{1}{v+1}\,dv$$

$$= \int\frac{1}{x}\,dx$$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$$\frac{v^2-1}{v} = Ax$$

$y = vx \;\therefore\; v = y/x$

$$\frac{(y/x)^2 - 1}{y/x} = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax \cdot \frac{y}{x}$$

$y^2/x^2 - 1 = Ay$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2y + x^2$

$y^2 = x^2(Ay + 1)$

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