Kien-Olali Kingsley 18/ENG05/027

1) $\frac{dy}{dx} = 2\sinh x - y\tanh x$; $\frac{dy}{dx} + y\tanh x = 2\sinh x$; $P = \tanh x$
$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

$\cosh x = u$

$\frac{du}{dx} = \sinh x$

$\int \frac{\sinh x}{u}\,dx$ d

$dx = du/\sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x} = \int \frac{1}{u}\,du = \ln u = \ln \cosh x$

$IF = e^{\int P\,dx} = e^{\ln \cosh x}$ ∴ $IF = \cosh x$

$y \cdot IF = \int Q\,IF\,dx$

$y\cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x\cos x = \sin(2x)$

∴ $2\sinh x\cosh x = \sinh(2x)$

$y\cosh x = \int \sin 2x\,dx$

$y\cosh x = \frac{1}{2}\cosh 2x + C$; $\cosh x\,y = \frac{\cosh 2x}{2} + C$

$y = \frac{\frac{\cosh 2x}{2} + C}{\cosh x}$; $y = \frac{\cosh 2x + 2C}{\cosh x}$; Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b)

$y/\frac{dy}{dx}$

b) $\frac{dy}{dx} + 2y = e^{3x}$; $P = 2$, $Q = e^{3x}$; $IF = e^{\int P\,dx}$
$\int P\,dx = 2x$

$IF = e^{2x}$

~~$y\,IF = e$~~ $y\,IF = \int Q\,IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = \frac{1}{5}e^{5x} + C$

$y = \frac{1}{5}e^{3x} + Ce^{-2x}$

c) $2\frac{dy}{dx} = x^2 + 2x - 3$; $\frac{dy}{dx} = x + 2 - \frac{3}{x}$; $\int \frac{dy}{dx}\,dx = \int x + 2 - \frac{3}{x}\,dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$; $\frac{dy}{dx}y^{-3} + \frac{y^{-2}}{x} = 1$ — (1) ; $z = y^{1-n}$ $z = y^{-2}$ — (2)

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ — (3) ; multiply (1) (3) (2) $-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$ — (4)

$\frac{dz}{dy} = \frac{-2y^{-3}\,dy}{dx}$; sub (2) & (3) into (4)

$\frac{dz}{dy} - \frac{2z}{x} = -2$

∴ $P = -2/x$ $Q = -2$; $\int P\,dx = -2\ln x$, $IF = e^{-2\ln x} = x^{-2}$

$z\,IF = \int Q\,IF\,dx$; $z \cdot x^{-2} = \int -2x^{-2}\,dx = -2\int x^{-2}\,dx$

$= -\frac{2x^{-1}}{-1} + C$ $= z \cdot x^{-2} = 2x^{-1} + C$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$; $z = 2x + Cx^2$; $z = y^{-2}$

∴ $y^{-2} = x(2 + Cx)$

e) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$; $\frac{dy}{dx} = x\sin 3x + 4/x^2$; $\int \frac{dy}{dx} = \int x\sin 3x + \int 4x^{-2}$

$= \frac{1}{3}(\cos 3x + 3\sin 3x/9 - 4x^{-1})$

f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$; $y = vx$; $\frac{dy}{dx} = v + x\frac{dv}{dx}$

~~$\frac{dy}{dx}$~~ $v + x\frac{dv}{dx} = 2(vx)^3/x^3 + v^2x^3$

$v + x\frac{dv}{dx} = 2v^3/x^3(1+v^2)$

$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v = \frac{2v^3 - v(1+v^2)}{1+v^2}$

$x\,dv = \frac{v^3 - v}{1+v^2}$; $\frac{1+v^2}{v^3 - v}\,dv =$

f ꝭ $x^3 + x(y^2)\, {dy}/{dx} = 2y^3$

$y = vx \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\frac{dy}{dx} = \frac{2y^3}{(x^3+xy^2)} = v + x\frac{dv}{dx} = \frac{2(vx)^3}{(x^3+x(vx)^2}$$

$$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3+v^2x^3} \quad ; \quad v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3 - (v+v^3)}{1+v^2} = x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

$$\frac{1+v^2}{v^3-v}\,dv = \frac{1\,dx}{x}$$

$$\int \frac{1+v^2}{v^3-v} = \int \frac{1}{x}\,dx = \int \frac{1+v^2}{v^3-v} = \int \frac{1}{x}\,dx$$

$$-\ln(v) + \ln(v-1) + \ln((v+1) \cancel{=} \ln x + c$$

$$= -\ln(\tfrac{y}{x}) + \ln(y/x - 1) + \ln(y/x + 1) = \ln x + c_1$$