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MATRIC NO 18/ENG01/016
CHEMICAL ENGINEERING.
ENGINEERING MATHS

a.) $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = 2\sinh x$

$\therefore P = \tanh x$

$Q = 2\sinh x$

$\int P \cdot dx = \int \tanh x - \int \frac{\sinh x}{\cosh x} \cdot dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x \qquad dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u} \cdot du$

$= \ln u$

$= \ln \cosh x$

$I.F = e^{\int P \cdot dx}$

$= e^{\ln(\cosh x)}$

$I.F = \cosh x$

Then, $y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \cdot dx$

Recall that,

$2\sin x \cos x = \sin(2x)$

$\therefore 2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh(2x) \cdot dx$

$y \cdot \cosh x = \frac{1}{2} \cdot \cosh 2x + C$

$\cosh + y = \frac{\cosh 2x}{2} + C$

$y = \frac{\frac{\cosh 2x}{2} + C}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

Let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b.) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2, \int P\,dx = 2x$

$Q = e^{3x}$

$\therefore I.F = e^{\int P\,dx}$

$= e^{2x}$

$\therefore y \cdot IF = \int Q \cdot I.F\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \cdot dx$

$y \cdot e^{2x} = \int e^{5x} \cdot dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$

$y = \frac{\frac{1}{5} e^{5x} + C}{e^{2x}}$

$y = \frac{1}{5} e^{3x} + Ce^{-2x}$

c.) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx} \cdot dx = \int x + 2 - \frac{3}{x} \cdot dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + A$

d) $\frac{dy}{dx} + \frac{y}{x} = y^2$

$\frac{dy}{dx} y^{-3} + \frac{y^{-2}}{x} = 1$

$z = y^{1-n}$.

$n = 3$

$z = y^{1-3}$

$z = y^{-2}$.

$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx}$

$\therefore$

$-2y^{-3} \frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$

and $\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx}$.

$\therefore$

$\frac{dz}{dy} - \frac{2z}{x} = -2$.

$P = -2/x$.

$Q = -2$.

$\int P dx = -2 \ln x$

$I.F = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF \, dx$.

$z \cdot x^{-2} = \int -2x^{-2} \cdot dx$

$= -2 \int x^{-2} \cdot dx$

$= \frac{-2x^{-1}}{-1} + C$

$z \cdot x^{-2} = 2x^{-1} + C$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$

$z = 2x + Cx^2$

$z = x(2 + Cx$

$z = y^{-2}$

$\therefore y^{-2} = x(2 + Cx)$

$y^2 = \frac{1}{x(2 + Cx)}$.

$y = \sqrt{\frac{1}{x(2+Cx)}} = \frac{1}{\sqrt{x(2+Cx}}$.

e) $x^2 \frac{dy}{dx} = x^3 \sin^3 x + 4$

$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$.

$\int \frac{dy}{dx} = \int x \sin 3x + \int 4x^{-2}$

$= \frac{1}{3}\cos 3x + \int \frac{1}{3}\cos 3x + \frac{4x^{-1}}{-1}$

$= \frac{-2\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$\therefore y = \frac{\sin 3x}{9} - \frac{x \cos 3x}{3} - \frac{4}{x}$.

F.) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\, dv/dx$.

$v + x\frac{dv}{dx} = \frac{2(vx)^2}{x^3 + v^2x^3}$

$v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^3 + v^2x^3}$

$x\frac{dv}{dx} = \frac{2v^2}{1+v^2} - v$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$