$$\frac{dx}{x} = \frac{1+V^2}{V^3-V}\,dv \quad V^3-V^2\,(V)(V-1)(V+1)$$

$$\frac{1+V^2}{V^3-V^2} = \frac{A}{V} + \frac{B}{V-1} + \frac{C}{V+1}$$

$$1+V^2 = A(V-1)(V+1) + B(V)(V+1) + C(V)(V-1)$$

$V=1$, $1+1^2 \; B(1)(2)$

$2 = B(2) \quad B = 1$.

$V=-1$ $\quad 1+(-1)^2 = C(-1)(-1-1)$

$2 = 2C \quad C = 1$.

$V=0$, $1+(0)^2 = A(0-1)(0+1) \quad 1 = A(-1)$

$A = -1$.

$$\int\left[-\frac{1}{V} + \frac{1}{V-1} + \frac{1}{V+1}\right]dV = \int \frac{1}{x}\,dx$$

$$= \int -\frac{1}{V}\,dV + \int \frac{1}{V-1}\,dV \int \frac{1}{V+1}\,dV = \int \frac{1}{x}\,dx$$

$$= -\ln V + \ln(V-1) + \ln(V+1) = \ln x + C.$$

$$= -\ln V + \ln(V-1) + \ln(V+1) = \ln x + \ln A.$$

$$\frac{V^2-1}{V} = Ax, \quad y = Vx \quad V = \frac{y}{x}.$$

$$\frac{(y/x)^2 - 1}{y/x} = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax \cdot \frac{y}{x}$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = x^2\,(Ay+1)$$

NAME AKA PEACE OTAOGHENE
18/ENG01/002
CHEMICAL ENGINEERING

a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x$, $Q = 2\sinh x$

$\int P dx = \int \tanh x \, dx = \int \frac{\sinh x}{\cosh x} dx$

$= \ln\cosh x$

$IF = e^{\ln\cosh x} = \cosh x$

$y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$y \cdot \cosh x = 2\int \sinh x \cdot \cosh x \, dx$

$y \cdot \cosh x = 2\int \cosh x \, d(\sinh x)$

$= 2\left[\frac{\cosh^2 x}{2}\right] + c$

$y \cdot \cosh x = \cosh^2 x + c$

$y = \cosh x + \frac{c}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2$, $Q = e^{3x}$

$\int P dx = \int 2 dx = 2x$

$IF = e^{2x}$

$y \cdot IF = \int Q \, IF \, dx$

$y \cdot e^{2x} = \int e^{3x} e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y e^{2x} = \frac{e^{5x}}{5} + c$

$y = \frac{e^{3x}}{5} + \frac{c}{e^{2x}}$

$y = \frac{e^{3x}}{5} + ce^{-2x}$

c) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = \frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x}$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x}$

$\int dy = \int x + 2 - \frac{3}{x} \, dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$ --(1)

$\frac{dy}{dx} + y \cdot x^{-1} = y^3$ $n = 3$

Divide both by y^3

$y^{-3}\frac{dy}{dx} + y^{-2}x^{-1} = 1$ --(2)

$z = y^{1-n}$

$z = y^{1-3}$, $z = y^{-2}$

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$

multiply eqn 2 by $y^{1-n} = y^{-2}$

$-2y^{-3}\frac{dy}{dx} + (-2y^{-2}x^{-1}) = -2$ --(3)

$-2y^{-3}\frac{dy}{dx} - 2y^{-2}x^{-1} = -2$ --(4)

Put $z = y^{-2}$ and $\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ in 4

$\frac{dz}{dy} - 2zx^{-1} = -2$

$\frac{dz}{dy} - \frac{2z}{x} = -2.$

$P = \frac{-2}{x}$, $Q = -2.$

$\int P\,dx = -2\ln x.$ $\quad IF = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF\,dx$

$z \cdot x^{-2} = \int -2 \cdot x^{-2}\,dx$

$z \cdot x^{-2} = -2\int x^{-2}\,dx$

$z \cdot x^{-2} = \frac{+2x^{-1}}{+1} + C$

$z \cdot x^{-2} = 2x^{-1} + C.$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$

$z = 2x + Cx^2$

$z = x(2 + Cx)$

$z = y^{-2}.$

$y^{-2} = x(2 + Cx)$

$y^2 = \frac{1}{x[2 + Cx]}$

$y = \sqrt{\frac{1}{x(2+x)}}$

e) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

$\frac{dy}{dx} = x\sin 3x + \frac{4}{x^2}.$

$\int\frac{dy}{dx} = \int x\sin 3x + \int 4x^{-2}$

$= \frac{1}{3}\cos 3x + \int\frac{1}{3}\cos 3x + \frac{4x^{-1}}{-1}$

$= -\frac{2\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{9} - \frac{x\cos 3x}{3} - \frac{4}{x}.$

f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$y = Vx.$

$\frac{dy}{dx} = V + x\frac{dy}{dx}$

$V + x\frac{dv}{dx} = \frac{2(Vx)^3}{x^3 + V^2x^3}$

$V + x\frac{dv}{dx} = \frac{\cancel{x^3}(2V^3)}{\cancel{x^3}(1+V^2)}$

$x\frac{dv}{dx} = \frac{2V^3}{1+V^2} - V$

$= \frac{2V^3}{1+V^2} - V$

$= \frac{2V^3 - V(1+V^2)}{1+V^2}$

$x\frac{dv}{dx} = \frac{2V^3 - V - V^3}{1+V^2}$

$= \frac{V^3 - V}{}$