Berema Precious

18/ENG03/021

Civil Engineering

a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x \quad dx = \frac{dy}{\sinh x}$

$\frac{du}{dx} = \sinh x \cdot \frac{dy}{\sinh x}$

$\int \frac{1}{u} du = \ln U = \ln \cosh x$

$If = e^{\int P dx} = e^{\ln \cosh x}$

$If = \cosh x$

Then $y \cdot If = \int Q \cdot If dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x dx$

$2\sinh x \cosh x = \sinh 2x$

$y \cdot \cosh x = \frac{1}{2} \times 2\cosh 2x + c$

$y\cosh x = \cosh 2x + c$

$y = \frac{\frac{\cosh 2x + c}{2}}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$If = e^{\int P dx} = e^{2x}$

$y \cdot If = \int Q \cdot If dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = \frac{e^{5x}}{5} + c$

$y = \left(\frac{e^{5x}}{5} + c\right) \div e^{2x}$

$y = \frac{e^{3x}}{5} + e^{-2x} \cancel{c}$

c) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - 3x^{-1}$

$\int \frac{dy}{dx} = \int x + 2 - 3x^{-1} dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

comparing to $\frac{dy}{dx} + Py = Qy^n$

$Q = 1$; $P = x^{-1}$ $\therefore n = 3$

divide through by y^3

$y^{-3}\frac{dy}{dx} + \frac{y^{-2}}{x} = 1 \quad \ldots\ldots (1)$

let $z = y^{-2}$; sub in eqn (1)

$= y^{-3}\frac{dy}{dx} + \frac{z}{x} = 1$

differentiate Z with respect to x

$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$ ...(1)

multiply through by eqn (1) by -2

$-2y^{-2}\frac{dy}{dx} + \frac{-2y^{-2}}{x} = -2$

Substitute $Z = y^{-2}$ and $\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$

In eqn (4)

$\frac{dz}{dx} = \frac{2Z}{x} = -2$ ...(3)

Applying integrating factor in eqn (3)

$\frac{dy}{dx} + Py = Q$

$P = \frac{-2}{x}$ $\quad Q = -2$

$\int P dx = -2\ln x$

$e^{\int P dx} = e^{-2\ln x} = IF$

$IF = \frac{1}{x^2}$

$Z \cdot IF = \int Q \cdot IF \, dx$

$Z \cdot \frac{1}{x^2} = \int -2 \cdot \frac{1}{x^2} dx$

$Z \cdot \frac{1}{x^2} = \int \frac{-2}{x^2} dx$

$Z \cdot \frac{1}{x^2} = \int -2x^{-2} dx$

$\frac{Z}{x^2} = \frac{-2x^{-2+1}}{-2+1} + c$

$\frac{Z}{x^2} = 2x^{-1} + c$

multiply through by x^2

$Z = 2x + x^2 c$

recall that $Z = y^{-2}$

$y^{-2} = 2x + x^2 c$

$y = \frac{1}{\sqrt{2x + x^2 c}}$

e) $x^2\frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = x \sin 3x + 4x^{-2}$

$\int \frac{dy}{dx} = \int x \sin 3x + \int 4x^{-2}$

$u = x$ $\quad dv = \sin 3x$

$du = 1$ $\quad v = -\frac{\cos 3x}{3}$

$uv - \int v\,du$

$= x \cdot -\frac{\cos 3x}{3} - \int -\frac{\cos 3x}{3}$

$-\frac{x\cos 3x}{3} - \frac{\sin 3x}{9}$

$= -\frac{x\cos 3x}{3} - \frac{\sin 3x}{9} - 4x^{-1} + c$

f) $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\, dv/dx$

$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} = \frac{2v^3 - v(1+v^2)}{1+v^2}$

$x\,dv = v^3$

$\frac{1+v^2}{v^3 - v} dv = \frac{1}{x} dx$

$v(v-1)(v+1) = v^3 - v$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$

$1 + 1^2 = B(1)(2)$

$2 = B(2)$ $\therefore B = 1$

$v = -1$; $1 + (-1)^2 = C(-1)(-1-1)$ $\therefore C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(2+1)$

$1 = A(-1)(1)$

$1 = A - 1$

$A = -1$

$\int -\frac{1}{v}$ ...

$= \int \frac{1}{x}$ ...

$-\ln v +$ ...

$\ln(v-1$ ...

$y = vx$

$$\int \left[\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv = \int dx \frac{1}{x}$$

$$\int \frac{-1}{v} dv + \int \frac{1}{v-1} dv + \int \frac{1}{v+1} dv$$

$$= \int \frac{1}{x} dx$$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$$

$$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$$

$$\frac{v^2-1}{v} = Ax$$

$$y = vx \quad : \quad v = y/x$$

$$\frac{(y/x)^2 - 1}{y/x} = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax \cdot \frac{y}{x}$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(Ay + 1)$$