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mechanical engineering.

solve the following differential equations.

(a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

(b) $\frac{dy}{dx} + 2y = 3e^{3x}$

(c) $2x\frac{dy}{dx} = x^2 + 2x - 3$

(d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

(e) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

(f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

solution

(a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$\cancel{\frac{dy}{dx} = 2\sinh x - y\tanh x}$

let $P = \tan x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x$

$dx = \frac{du}{\sinh x}$

$\therefore \int \frac{\sinh x}{u} dx = \int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\therefore \int \frac{1}{u} du = \ln u$

recall $u = \cosh x$

$\therefore \ln \cosh x$

I.f

$= e^{\int P dx}$

$\therefore e^{\int P dx} = e^{\int \ln \cosh x} = \cosh x$

$\therefore IF = \cosh x$

$y \cdot If = \int Q \cdot If \, dx$

$y \cosh x = \int Q \cdot \cosh x \, dx$

$y \cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$\int 2\sinh x \cdot \cosh x \, dx = 2\sin x \cos x = \sinh(2x)$

$\therefore y\cosh x = \int \sinh(2x)$

$y\cosh x = \frac{1}{2}\cosh 2x + C$

$\therefore y = \frac{\cosh 2x + C}{\cosh x}$

(B) $\frac{dy}{dx}$

I

(5) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2, Q = e^{3x}$

$If = e^{\int P dx}$

$\int P dx = \int 2 dx = 2x$

$IF = e^{\int P dx} = e^{2x}$

$y \cdot If = \int Q \cdot If \, dx$

$ye^{2x} = \int Q \cdot e^{2x} dx$

$\int Q \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$= \int e^{5x}$

$\therefore ye^{2x} = \int e^{5x} dx$

$ye^{2x} = \frac{1}{5} e^{5x} + c$

$\therefore y = \frac{e^{5x} + c}{5e^{2x}}$

(c) $2\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 \cdot \frac{3}{x}$

$\int \frac{dy}{dx} = \int x + 2 \cdot \frac{3}{x} \, dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

(d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} y^{-3} + \frac{y^{-3}}{x} = 1 \quad -(1)$

$z = y^{1-n}$

$n = 3$

$z = y^{-2} \quad -(2)$

$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx} \quad -(3)$

multiplying (1) by (3)

$\therefore -2y^{-3} \frac{dy}{dx} - \frac{2y^{-2}}{x} = -2 \quad -(4)$

and $\frac{dz}{dy} = \frac{-2y^{-3} dy}{dx}$

~~multiplying~~

Substitute eqn (2) and (3) into (4)

$\frac{dz}{dy} - \frac{2x}{x} = -2$

$P = \frac{-2}{x}$

$Q = -2$

$\int P dx = -2 \ln x$

$IF = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF dx$

$zx^{-2} = \int -2x^{-2} dx$

$y^{1-n} x^{-2} = 2x^{-1} + c$

$y^{1-n} = \frac{2x^{-1} + c}{x^{-2}}$

$y^{-2} = \frac{2x^{-1} + c}{x^{-2}}$

(3) $x^2 \frac{dy}{dx} = x^3 \sin^3 x + 4$

$\frac{dy}{dx} = x \sin 3x + 4x^{-2}$

$\int \frac{dy}{dx} = \int x \sin 3x + \int 4x^{-2}$

$= \frac{1}{3} \cos 3x + \frac{\sin 3x}{9} - 4x^{-1}$

(5) $(x^3 + xy^2) \frac{dy}{dx} = 2y^3$

$\cancel{\frac{dy}{dx} = \frac{2y^3}{x^3 + 2y}}$ $\quad y = vx$

$\frac{dy}{dx} = v + x \frac{dv}{dz}$